Properties of voting systems: Homework

Mathematics 170

due Tuesday, April 22

1. *Mathematics and Politics*, p. 131, problem 12, parts (a) and (c).

2. *Mathematics and Politics*, pp. 131–132, problems 18 and 19.

   Hint for problem 18:

   The following election can be used to give counterexamples for the Coombs system in parts (c), (d), and (e).

   Consider three candidates A, B, and C, and five voters with preferences

A	B	B	C	B
C	C	A	A	C
B	A	C	B	A

3. *Mathematics and Politics*, pp. 132, problem 20. (Hint: Note that the "top condition" can be rephrased as: "If a candidate is not on top of any preference list, then the candidate is not the winner." So your counterexamples should all involve candidate who receive no first-place votes and yet win the election.)

4. Let us define a voting system called "simulated runoff." The idea is to simulate the results of a genuine runoff election between the top two vote-getters. In simulated runoff, the winner of the election is the winner of the pairwise contest between the candidates who got the highest and second-highest number of first-place votes. (This is what would happen if a real runoff election were held.)

   (a) Explain why in a three-person election, instant runoff (the Hare system) and simulated runoff will always produce the same winners.

   (b) Show that in the following four-person election, instant runoff and simulated runoff produce different winners.

35%	**25%**	**20%**	**10%**	**10%**
A	C	B	B	D
B	A	C	A	C
C	B	A	C	A
D	D	D	D	B