

Answers to Voting Systems Quiz

Question 1. Use the following election to prove that sequential pairwise voting does not satisfy the Pareto condition. The agenda is Jospin, Bayrou, Chirac, Le Pen.

- Jospin, Bayrou, Le Pen, Chirac: 35%
- Jospin, Bayrou, Chirac, Le Pen: 10%
- Chirac, Jospin, Bayrou, Le Pen: 15%
- Chirac, Bayrou, Jospin, Le Pen: 20%
- Bayrou, Le Pen, Chirac, Jospin: 20%

Answer The Pareto condition says that if every voter prefers candidate A over candidate B , then candidate B is not a winner. So to show Pareto is not satisfied, we notice that every voter prefers Bayrou over Le Pen. Then we show that Le Pen is the winner.

To do this, we first pair Jospin against Bayrou, and find that Jospin wins by 60–40. Then we pair Jospin against Chirac, and find that Chirac wins by 55–45. Finally we pair Chirac against Le Pen, and find that Le Pen wins by 55–45. So Le Pen is the winner in sequential pairwise voting, even though everyone unanimously prefers Bayrou.

So sequential pairwise voting does not satisfy the Pareto condition.

Question 2. Consider the following election:

- Carter, Anderson, Reagan: 42%
- Carter, Reagan, Anderson: 4%
- Reagan, Carter, Anderson: 4%
- Reagan, Anderson, Carter: 43%
- Anderson, Carter, Reagan: 4%
- Anderson, Reagan, Carter: 3%

(a) Find the winner if the Borda count is used.

Answer We give two points to the first-place candidate, one point to the second-place candidate, and zero points to the third-place candidate. Then the numbers are:

- **Carter:** $42 \times 2 + 4 \times 2 + 4 \times 1 + 43 \times 0 + 4 \times 1 + 3 \times 0 = 84 + 8 + 4 + 4 = 100$
- **Reagan:** $42 \times 0 + 4 \times 1 + 4 \times 2 + 43 \times 2 + 4 \times 0 + 3 \times 1 = 4 + 8 + 86 + 3 = 101$
- **Anderson:** $42 \times 1 + 4 \times 0 + 4 \times 0 + 43 \times 1 + 4 \times 2 + 3 \times 2 = 42 + 43 + 8 + 6 = 99$

So Reagan is the winner.

(b) Find the winner if instant runoff (the Hare system) is used.

Answer Since no candidate has over 50% of the first-place votes, we eliminate the candidate with the fewest first-place votes. This is clearly Anderson. Then we consider the race between Carter and Reagan. Carter ends up with $42 + 4 + 4 = 50\%$ of the first-place vote, while Reagan gets $4 + 43 + 3 = 50\%$ of the first-place vote. So Reagan and Carter are tied for the win.

Question 3. Let us define a voting system called “Ostracize.” In this system, we declare the candidate(s) with the most last-place votes to be the loser(s), and all remaining candidates to be tied for the win.

Consider the following election.

- Taft, Wilson, Roosevelt: 25%
- Roosevelt, Taft, Wilson: 29%
- Wilson, Roosevelt, Taft: 45%

Use this election to show that “Ostracize” does not satisfy independence of irrelevant alternatives.

Answer The result of the election is that Taft loses (since he has 45% of the last-place votes), while Wilson and Roosevelt tie for the win.

But notice that if Roosevelt drops out of the race, then the votes become

- Taft, Wilson: 54%
- Wilson, Taft: 45%

So Wilson has the most last-place votes and thus he is the loser and Taft is the winner.

But independence of irrelevant alternatives means that a pairwise loser cannot win the election, and since Wilson is a pairwise loser, his victory in the three-way race means IIA is not satisfied by “Ostracize.”