Math 240, Fall 2004: Review Sheet for Second Midterm

Be able to:

- Find a tangent vector and tangent line to a vector function of time.
- Compute the length of a curve given as a vector function.
- Determine the curvature of a curve at any point.
- Find the tangent $\mathbf{T}$, normal $\mathbf{N}$, and binormal $\mathbf{B}$ vectors of a curve at any point.
- Compute the tangential and normal components of acceleration.
- Compute divergence and curl of any three-dimensional vector field.
- Compute any of the three types of line integrals.
- Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for any curve C and vector field $\mathbf{F}$.
- Determine whether $\int_C \mathbf{F} \cdot d\mathbf{r}$ depends only on the endpoints of C; if so, find a potential function ϕ for $\mathbf{F}$.
- Apply Green's Theorem to
  - compute the circulation of a vector field around a closed curve.
  - rewrite a line integral in terms of a simpler one on a different curve.
- Compute surface area of a given surface.
- Compute the flux $\int_S \mathbf{F} \cdot \mathbf{n}\, dS$ given a force field $\mathbf{F}$ and a surface S.
- Apply Stokes' Theorem to compute the circulation of a vector field around a closed curve.
- Apply the Divergence Theorem to
  - compute flux through a closed surface.
  - rewrite a surface integral in terms of a simpler one on a different surface.
- Compute double integrals in any coordinate system, using the change-of-variables formula.

Things to understand:

- The different formulas for curvature and the normal vector, depending on whether the curve is parametrized by arc length or not.
- How to prove general vector calculus identities like $\operatorname{curl} \operatorname{grad} f = 0$. (Use the most general function or vector field possible, *never* a special one you make up.)
- The connection between Green's Theorem and Stokes' Theorem. (Green's is just a special case.)
- When Green's, Stokes', and the Divergence Theorem do *not* apply. (If the vector field has a singularity in the region.)
- The difference between Stokes' and the Divergence Theorem. (One goes from curves with no boundary to surfaces with boundary, the other goes from surfaces with no boundary to volumes with boundary.)
- How both Stokes' Theorem and the Divergence Theorem are analogous to the Fundamental Theorem of Calculus.

- The role of *orientation* in both Stokes' Theorem and the Divergence Theorem. (For Stokes', which direction to follow the curve and which way to point the normal; for Divergence, which way to point the normal.)

Old (Math 114) material you may need to review:

- The definition of a gradient, and how one uses it to find a normal vector $\mathbf{n}$ to a surface $g(x, y, z) = C$ or $z = f(x, y)$.
- Partial derivatives, especially the equality of mixed partials: $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- How to compute the potential function for an exact differential. (i.e., solve $\frac{\partial \phi}{\partial x} = P$, $\frac{\partial \phi}{\partial y} = Q$.)
- How to compute double integrals in Cartesian coordinates, or in polar coordinates. (Be able to set up your own limits.)
- How to compute triple integrals in Cartesian coordinates, or in cylindrical coordinates. (Be able to set up your own limits.)

Some important formulas (you may need more than these):

- The curvature formula, in both arc-length-parametrization form and otherwise.
- The definition of $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$.
- The normal component and tangential component of acceleration. (In terms of cross products, and also in terms of curvature and speed.)
- The formulas for divergence and curl.
- The arc length element ds and the surface area element dS.
- The exact statements of Green's, Stokes', and the Divergence Theorem.
- The Jacobian (change of variables formula).

You should be able to do the following review exercises:

- (conceptual) p. 558: 1–2, 4, 7–11, 13–14, 16, 18–20.
- (computational) pp. 558–560: 21–24, 26, 33, 39–65.