Math 240, Fall 2004: Review Sheet for Final Exam

Be able to:

- Recognize and solve a first-order nonlinear homogeneous differential equation, of the form $y' = f(y/x)$.
- Recognize and solve a Bernoulli differential equation, of the form $y' + P(x)y = f(x)y^n$.
- Recognize and solve a Cauchy-Euler equation, of the form $x^2y'' + pxy' + qy = 0$.
- Solve an eigenvalue problem like $y'' + \lambda y = 0$ with boundary conditions.
- Compute a Laplace transform of a simple $f(t)$ directly from the definition.
- Compute inverse Laplace transforms of rational functions, using partial fractions and your own table.
- Use Laplace transforms to solve differential equations with constant coefficients.
- Find a power series for the solution of a second-order differential equation about an ordinary point. Be able to find a formula for the coefficients in simple cases, or at least to write the first few terms in more complicated cases.
- Recognize a regular singular point, and find at least one power series solution about such a point using the method of Frobenius.
- Solve a system of the form $X' = AX$ where A is a constant matrix, using the method of eigenvectors.
- Solve a system of the form $X' = AX + F(t)$, using the variation of parameters formula.

Things to understand:

- When a point is a singular point of a differential equation. (e.g., why is $y' + xy = 0$ not singular at $x = 0$ but $xy' + y = 0$ is?)
- The general facts about solutions of differential equations:
  - There are n parameters in the general solution of an n^{th}-order differential equation.
  - If the differential equation is not singular at x_0, then these n parameters can be determined by the initial conditions at x_0.
  - If the differential equation is singular at x_0, then solutions can only be defined on one side of x_0, and solutions will generally either go to zero or infinity as x approaches x_0.
- That a system of n first-order differential equations is theoretically equivalent to a single n^{th}-order differential equation.
- Why some functions (e.g. $f(t) = e^{t^2}$) do not have Laplace transforms.

Old (Math 114) material you may need to review:

- How to solve first-order separable differential equations and first-order linear differential equations.
- How to solve second-order homogeneous linear differential equations with constant coefficients.
- How to use the method of undetermined coefficients to solve nonhomogeneous linear differential equations with constant coefficients.

Some important formulas (you may need more than these):

- The general solution of a linear first-order differential equation.
- The known substitutions that simplify first-order nonlinear differential equations.
- The formulas for the general solution of a linear second-order differential equation, with constant coefficients or in Cauchy-Euler form, in terms of the roots of the characteristic equation.
- Laplace transforms of elementary functions (the table in Theorem 4.1 is sufficient).
- The form of the power series in the method of Frobenius, and perhaps the indicial equation.
- The solution of a system by diagonalization, and the variation of parameters formula for a system.

You should be able to do the following review exercises:

- (conceptual)
  - (Chap. 2) p. 98: 2.
  - (Chap. 3) p. 186: 1.
  - (Chap. 4) p. 232: 4–6, 21.
  - (Chap. 5) p. 266: 5, 6.
  - (Chap. 10) p. 599: 1–4.
- (computational)
  - (Chap. 2) pp. 98–99: 8(a,c,e,f,h,i,j,m), 11, 14, 20.
  - (Chap. 3) pp. 187–188: 25–28, 51.
  - (Chap. 4) pp. 232–233: 7, 9, 13, 14, 16.
  - (Chap. 5) pp. 266: 7–10, 13–16.
  - (Chap. 10) pp. 599–600: 5–14.