Math 501 # 8 Shiying

1. For (a) & (b)

Let $\vec{t}, \vec{n}, \vec{b}$ be the Frenet frame of the curve.

geodesic: $\vec{n} = \pm N$

where $\vec{N}$ is the normal of the surface.

We have $\vec{n}'(s) = -\kappa(s)\vec{t}(s) + \tau(s)\vec{b}(s)$

For geodesics $\vec{N}'(s) = \pm\kappa(s)\vec{t}(s) \mp \tau(s)\vec{b}(s)$

Therefore line of curvature

$\Leftrightarrow$ $\vec{N}'(s) = \pm\kappa(s)\vec{t}(s)$ $\forall s$

$\Leftrightarrow$ $\tau(s) = 0$ $\forall s$

$\Leftrightarrow$ plane curve

(c) Let $S = \mathbb{R}^2$ & pick any non-straight curve on it.

5. It's not hard to see the max/min parallels are geodesics and lines of curvature. The upper parallel is asymptotic & line of curvature.

6. normal of upper parallel $\perp$ normal of torus

$\Rightarrow \kappa_g = \kappa = 1/\text{radius} = 1/a$

7. Let $\vec{e}_1 = \vec{i}$ & $\vec{e}_2 = \cos\theta\,\vec{j} + \sin\theta\,\vec{k}$

where $\vec{i}, \vec{j}, \vec{k}$ are the x, y, z basis in $\mathbb{R}^3$.

$\vec{e}_1$ & $\vec{e}_2$ span the plane in question. In $\{\vec{e}_1, \vec{e}_2\}$ frame, a (x_1, x_2) vector is $x_1\vec{e}_1 + x_2\vec{e}_2 = (x_1, x_2\cos\theta, x_2\sin\theta)$ in the $\vec{i}, \vec{j}, \vec{k}$ frame. Therefore, the curve C has eqn $x_1^2 + (x_2\cos\theta)^2 = 1$. It's an ellipse with $a = 1$, $b = 1/\cos\theta$.

At the intersection of C with $\vec{e}_2$ axis, it's not hard to calculate that $|K| = \left| x_2''(x_1)\big|_{x_1=0} \right| = 1/\cos\theta$

Since the normal of the curve has θ angle with the normal of the surface $|k_g| = \sin\theta / \cos\theta = \tan\theta$

For the other intersection $|k_g| = 0$ since the normals align.

12. Let W_1 & W_2 are the ~~tangent~~ vectors of these geodesics. By definition

$$\left[\frac{DW}{du}\right] = 0 \text{ which by prop 3 of do Carmo } = -\frac{E_v}{2\sqrt{EG}} \Rightarrow E_v = 0$$

Similarly $G_u = 0$

We can the refine $\tilde{u} = \int \frac{1}{\sqrt{E(u)}}\, du$ & $\tilde{v} = \int \frac{1}{\sqrt{G(v)}}\, dv$

where the integration is done on 1