

# Exam 1

Part I:

B B C B C B C C

Problem 9:

$$(a). \sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\overline{\sin z} = \frac{e^{-i\bar{z}} - e^{i\bar{z}}}{-2i} = \frac{e^{i\bar{z}} - e^{-i\bar{z}}}{2i} = \sin \bar{z}$$

$$(b). \text{ First } \tan z = \frac{\sin z}{\cos z}$$

$$\text{Since } \overline{\cos z} = \frac{\overline{e^{iz} + e^{-iz}}}{2} = \frac{e^{-i\bar{z}} + e^{i\bar{z}}}{2} = \cos \bar{z}$$

$$\overline{\tan z} = \frac{\overline{\sin z}}{\overline{\cos z}} = \frac{\sin \bar{z}}{\cos \bar{z}} = \tan \bar{z}$$

Problem 10:

$$f(z) = \frac{z^2 + iz + 1}{z^2 + iz^3} = \frac{z^2 + (iz + 1)}{z^2(iz + 1)}$$
$$= \frac{1}{iz + 1} + \frac{1}{z^2}$$

$$\oint_C f(z) dz = \oint_C \frac{1}{z^2} dz + \oint_C \frac{1}{iz + 1} dz$$
$$= 0 + \frac{1}{i} \oint_C \frac{1}{z - i} dz$$
$$= 0 + 2\pi i / i = 2\pi$$

Problem 11:

$$\int_C z \cos z dz = z \sin z + \cos z \Big|_{1+i}^i$$

$$= i \sin i + \cos i - (1+i) \sin(1+i) - \cos(1+i)$$

$$= -\sinh 1 + \cosh 1 - (1+i) (\sin 1 \cos i + \sin i \cos 1)$$
$$- (\cos 1 \cos i - \sin 1 \sin i)$$

$$= -\sinh 1 + \cosh 1 - (1+i) (\sin 1 \cosh 1 + i \sinh 1 \cos 1)$$

$$- \cos 1 \cosh 1 + i \sin 1 \sinh 1$$

$$= -\sinh 1 + \cosh 1 - \sin 1 \cosh 1 - e^{-1} \cos 1$$

$$- i (\sin 1 e^{-1} + \sinh 1 \cos 1)$$

Problem 12.

$$(a). \quad v = x^2 - y^2 \Rightarrow \begin{aligned} \partial_x v &= 2x \\ \partial_y v &= -2y \end{aligned}$$

$$\Rightarrow \partial_x^2 v + \partial_y^2 v = 2 - 2 = 0 \quad \checkmark$$

$$(b). \quad f = u + iv$$

$$\partial_x u = \partial_y v = -2y \Rightarrow u = -2xy + h(y)$$

$$\begin{aligned} \partial_y u = -\partial_x v = -2x &\Rightarrow -2x + h'(y) = -2x \\ &\Rightarrow h(y) = \text{const.} \end{aligned}$$

$$\Rightarrow f = -2xy + i(x^2 - y^2) + C$$

$$= i z^2 + C, \quad \text{where } C \text{ is real constant.}$$