17.6

12: $\dfrac{e^{2+3\pi i}}{e^{-3+\pi i/2}} = e^{5+\pi i/2} = i e^5$

14: $e^{2\bar{z}} = e^{2(x-yi)} = e^{2x}(\cos 2y - i \sin 2y)$

16: $e^{1/z} = e^{\frac{x-yi}{x^2+y^2}} = e^{\frac{x}{x^2+y^2}}\left(\cos \frac{y}{x^2+y^2} - i \sin \frac{y}{x^2+y^2}\right)$

20: $(e^z)^n = \left(e^x (\cos y + i \sin y)\right)^n$

$= e^{nx}(\cos ny + i \sin ny) = e^{nx+iny}$

$= e^{nz}$

22: Cauchy-Riemann equations are satisfied for e^{z^2} everywhere on the complex plane.

Or, $\partial_{\bar{z}} f^{z^2} = 0$ at any point.

30: $\mathrm{Ln}(-e^3) = \mathrm{Ln}(e^{3+\pi i}) = 3 + \pi i$

46: $3^{i/\pi} = e^{i/\pi(\ln 3 + 2n\pi i)}$

$= e^{-2n\pi} \cdot e^{i \ln 3/\pi}$

48: $(i^i)^2 = \left[e^{i(i\pi/2 + i \cdot 2n\pi)}\right]^2$

$= \left[e^{-(\pi/2 + 2n\pi)}\right]^2$

$= e^{-(1+4n)\pi}$

$i^{2i} = e^{2i(i\pi/2 + i2n\pi)}$

$= e^{-(1+4n)\pi}$

$(i^2)^i = (-1)^i = e^{i(\pi i + 2n\pi i)}$

$= e^{-(1+2n)\pi}$

50: skipped. It's straightforward.

17.7. 14: $\cos\left(\frac{\pi}{2} + i\ln 2\right)$

$$= \frac{1}{2}\left[e^{i(\pi/2 + i\ln 2)} + e^{-i(\pi/2 + i\ln 2)}\right]$$

$$= \frac{1}{2}\left[e^{-\ln 2}\cdot e^{i\pi/2} + e^{\ln 2}\cdot e^{-i\pi/2}\right]$$

$$= \frac{1}{2}\left(\frac{1}{2}\cdot i + 2\cdot(-i)\right)$$

$$= -\frac{3}{4} i$$

20. $\cos z - i\sin z = 0$

$\Rightarrow$ $e^{-iz} = 0$ no solution.

32. $$\overline{\sin z} = \overline{\frac{e^{iz} - e^{-iz}}{2i}} = \frac{e^{-i\bar{z}} - e^{i\bar{z}}}{-2i}$$

$$= \sin\bar{z}$$

$$\overline{\cos z} = \overline{\frac{e^{iz} + e^{-iz}}{2}} = \frac{e^{-i\bar{z}} + e^{i\bar{z}}}{2}$$

$$= \cos\bar{z}$$

18.1 2: $\int_C (2\bar{z} - z)\,dz$

$$= \int_0^2 [-t - 3i(t^2+2)](-1 + i \cdot 2t)\,dt$$

$$= \int_0^2 \left[t + 6t(t^2+2) + i(3t^2+6-2t^2)\right] dt$$

$$= \int_0^2 (6t^3 + 13t) + i(t^2 - 6)\,dt$$

$$= \frac{3}{2}t^4 + \frac{13}{2}t^2 + i\left(\frac{t^3}{3} - 6t\right)\Bigg|_0^2$$

$$= 24 + 26 - i\left(\frac{8}{3} - 12\right)$$

$$= 50 - \frac{28}{3}i$$

6: $\int_C |z|^2\,dz$

$$= \int_1^2 \left(t^4 + \frac{1}{t^2}\right)\left(2t - \frac{i}{t^2}\right) dt$$

$$= \int_1^2 \left[2t^5 + \frac{2}{t} - i\left(t^2 + \frac{1}{t^4}\right)\right] dt$$

$$= \frac{1}{3}t^6 + 2\ln t - i\left(\frac{t^3}{3} + \frac{1}{3}t^{-3}\right)\Bigg|_1^2$$

$$= 21 + 2\ln 2 - i\left(\frac{7}{3} + \frac{7}{24}\right)$$

$$= 21 + 2\ln 2 - i\,\frac{21}{8}$$

12: $\int_C \sin z \, dz$

$$= \int_0^1 \sin x \, dx + \int_0^1 \sin(1+iy)\, i \, dy$$

$$= 1-\cos 1 + \frac{1}{2}\int_0^1 \left[e^{i(1+iy)} - e^{-i(1+iy)} \right] dy$$

$$= 1-\cos 1 + \frac{1}{2}\left[e^{i}(1-e^{-1}) + e^{-i}(e-1) \right]$$

$$= 1-\cos 1 + \cos 1 - \frac{e^{i-1}+e^{1-i}}{2}$$

$$= 1-\cos(1+i)$$

20: $\oint_C \bar{z}^2 \, dz$

$$= \int_0^1 x^2 \, dx + \int_0^1 (1-y^2-2yi)\, i \, dy$$

$$+ \int_1^0 t^2(-2i)(1+i)\, dt$$

$$= \frac{1}{3} + i\left(1-\frac{1}{3}\right) + 1 + (2i-2)\cdot\frac{1}{3}$$

$$= 2/3 + i \cdot 4/3$$

26: $\left| \int_C \frac{1}{z^2-2i} \, dz \right| \leq \text{Max} \frac{1}{|z^2-2i|} \cdot \text{Length of } C$

$$= \frac{6\pi}{\min|z^2-2i|} \leq \frac{6\pi}{\left||z^2|-|2i|\right|} = \frac{6\pi}{34}.$$

28. $\left| \int_C \frac{1}{z^3}\, dz \right| \leq \text{Max} \frac{1}{|z^3|}$ Length of C

$= \frac{1}{4^3} \cdot \frac{\pi}{2} \cdot 4 = \frac{\pi}{32}$

34. $\oint_C \bar{f}\, dz$

$= \oint \frac{1}{z-1}\, dz$

let $z = 1 + e^{i\theta}$

$= \int_0^{2\pi} e^{-i\theta} \cdot e^{i\theta} \cdot i\, d\theta$

$= 2\pi i$

circulation $= \text{Re}(2\pi i) = 0$

net flux $= \text{Im}(2\pi i) = 2\pi$