18.4. 2. $C: |z|=4$

$$\oint_C \frac{z^2}{(z-3i)^2}\,dz = 2\pi i \cdot 2z\Big|_{z=3i} = 12\pi i$$

14. $C: |z-1|=3$

$$\oint_C \frac{e^{-z}\sin z}{z^3}\,dz = 2\pi i \ \frac{d^2}{dz^2}\left(e^{-z}\sin z\right)\Big|_{z=0}$$

$$= 2\pi i \cdot \left(e^{-z}\sin z - 2e^{-z}\cos z - e^{-z}\sin z \Big|_{z=0} \right)$$

$$= -4\pi i$$

18. (a) $C: |z|=1$

$$\oint_C \frac{1}{z^3(z-4)}\,dz = 2\pi i \cdot \frac{d^2}{dz^2}\ \frac{1}{z-4}\Big|_{z=0}$$

$$= 2\pi i\ (-1)\cdot(-2)\ \frac{1}{(z-4)^3}\Big|_{z=0}$$

$$= \pi i/16$$

(b) $C: |z-2|=1$

No poles. $\oint_C \frac{1}{z^3(z-4)}\,dz = 0.$

19.1.6. $\lim_{n\to\infty} \frac{ni+2^n}{3ni+5^n} = \lim_{n\to\infty} \left(\frac{2}{5}\right)^n \frac{1+2^{-n}\cdot ni}{1+5^{-n}\cdot 3ni}$

$= 0 \cdot \frac{1}{1} = 0.$

$\left\{\frac{ni+2^n}{3ni+5^n}\right\}$ converges to 0.

18. $\sum_{k=0}^{\infty} \frac{1}{2} i^k = \frac{1}{2} \sum_{k=0}^{\infty} i^k$

$= \frac{1}{2} \sum_{p=0}^{\infty} (1+i-1-i)\, i^{4p}$

$= 0.$

22. radius of convergence R

$= \lim_{k\to\infty} \left| \frac{1}{k}\left(\frac{i}{1+i}\right)^k \Big/ \frac{1}{k+1}\left(\frac{i}{1+i}\right)^{k+1} \right|$

$= \lim_{k\to\infty} \frac{k+1}{k} \cdot \left|\frac{1+i}{i}\right|$

$= \sqrt{2}$

2b.

Since $\lim_{k\to\infty} \left(\frac{1}{k^k}\right)^{1/k} = \lim_{k\to\infty} \frac{1}{k} = 0$

$R = \infty$