

19.2.20.

$$\sin z = \sin\left(z - \frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$= \cos\left(z - \frac{\pi}{2}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k!} \left(z - \frac{\pi}{2}\right)^{2k}$$

$R = \infty$ since $\sin z$ is entire.

$$22. (z-1)e^{-2z} = (z-1)e^{-2(z-1)} \cdot e^{-2}$$

$$= e^{-2} (z-1) \sum_{n=0}^{\infty} \frac{(-2)^n}{n!} (z-1)^n$$

$$= \sum_{n=0}^{\infty} \frac{e^{-2}}{n!} (-2)^n (z-1)^{n+1}$$

$R = \infty$ since $(z-1)e^{-2z}$ is entire.

$$30. f(z) = \frac{1}{z}$$

$$\text{at } z_0 = 1+i \quad f(z) = \frac{1}{1+i} \frac{1}{1 + \frac{z-1-i}{1+i}} = \frac{1}{1+i} \sum_{n=0}^{\infty} (-1)^n \left(\frac{z-1-i}{1+i}\right)^n$$
$$= \sum_{n=0}^{\infty} (-1)^n (1+i)^{-n-1} (z-1-i)^n$$

$$\text{at } z_0 = 3 \quad f(z) = \frac{1}{3} \frac{1}{1 + \frac{z-3}{3}} = \frac{1}{3} \sum_{n=0}^{\infty} (-1)^n \left(\frac{z-3}{3}\right)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n 3^{-n-1} (z-3)^n$$

