

## GEOMETRY HW 3

CLAY SHONKWILER

1

If  $\sigma : E \rightarrow B$  is a covering with deck group  $D$ , show that  $D$  operates properly discontinuously and freely on  $E$ .

2

If  $D$  acts properly discontinuously and freely on  $E$ , show that  $\sigma : E \rightarrow E/D$  is a covering with deck group  $D$ .

3

If  $E \rightarrow B$  is a covering with  $B$  a manifold, show that  $E$  is a manifold also. I relieve you of the duty to show that  $E$  is also second countable. If  $E$  is the universal cover of  $B$ , show that  $\pi_1(B)$  acts differentiably, properly discontinuously and freely on the manifold  $E$ .

4

If  $\sigma : E \rightarrow G$  is a covering with  $G$  a Lie group, show that  $E$  is a Lie group also, the multiplication is unique if we choose an identity element over the identity element in  $G$ , and  $\sigma$  is a homomorphism.

5

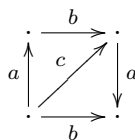
True or false: A map  $\sigma : E \rightarrow B$  which has the unique path lifting property must be a covering.

6

Find an example of a (non-trivial) covering where the deck group is trivial.

7

Compute the simplicial homology of the Klein bottle and  $S^2$  for an appropriate  $\Delta$  complex.



1

Let  $X$  be obtained by starting with two 2-simplices,  $[v_0, v_1, v_2]$  and  $[w_0, w_1, w_2]$  and identifying the corresponding vertices:

$$X = [v_0, v_1, v_2] \sqcup [w_0, w_1, w_2] / \{v_i \sim w_i\}.$$

What are the simplicial homology groups of  $X$ ?

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