

DIFFERENTIAL GEOMETRY HW 6

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- (1) Is it possible to make the double torus (i.e. orientable surface of genus 2) into a Lie group?
- (2) Give an example of a Riemannian metric on a Lie group which is not bi-invariant. Is it possible to put a bi-invariant metric on this Lie group?
- (3) Describe the geodesics in $SO(n)$.
- (4) Is the product metric on $S^3 \times S^1$ bi-invariant?
- (5) What are the Gauss-Kronecker and mean curvatures of an immersed hypersurface N of a manifold M ? Show that if the Gauss-Kronecker curvature vanishes somewhere and M is hyperbolic space then N cannot be flat.
- (6) What does it mean for a submanifold to be minimal? Give an example of a minimal submanifold of \mathbb{R}^3 .
- (7) Can you put a metric on $S^2 \subset \mathbb{R}^3$ so that the identity inclusion is geodesic at a point $p \in S^2$? Is there a metric on S^2 so that it is totally geodesic in \mathbb{R}^3 ?
- (8) Let $p \in \mathbb{C}\mathbb{P}^2$ and consider the 2-sphere S^2 which is the cut locus of p . Is S^2 a minimal submanifold of $\mathbb{C}\mathbb{P}^2$? Is it totally geodesic?
- (9) Is M minimal in TM ? Totally geodesic?
- (10) Suppose N is a submanifold of M such that TN is a totally geodesic submanifold of TM . Is N totally geodesic in M ? What about the converse?
- (11) Find a totally geodesic submanifold of $SO(5)$ of dimension ≥ 2 .
- (12) What is the conjugate locus of a point $p \in S^2 \times S^2$?
- (13) Describe the focal locus of the catenoid.
- (14) If M and N are Riemannian manifolds, is “ M is locally isometric to N ” a symmetric relation?
- (15) Is it possible for a submanifold $N \subset M$ to be its own focal locus?

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