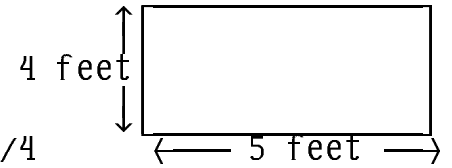


**Math 115 (Powers) Test. Thursday March 23, 2006 (with Answers)**

1. Two points  $(x,y)$  are chosen at random on a rectangle 4 feet by 5 feet. What is the probability that the two points are within a foot of each other? (i.e. compute  $\Pr(|x - y| < 1 \text{ foot})$ ).



Prob =

- A.  $1/3$  B.  $1/2$  C.  $3/8$  D.  $11/20$  E.  $2/3$  F.  $5/8$  G.  $7/30$  H.  $3/4$   
 ##

2. What is the variance of a random variable distributed on the interval  $[0,1]$  with probability density function  $f(x) = 2x$  for  $0 \leq x \leq 1$ ? Variance =  
 A.  $1/30$  B.  $1/18$  C.  $1/9$  D.  $2/9$  E.  $1/3$  F.  $1/6$  G.  $1/4$  H.  $1/2$   
 ##

3. A die is numbered 1,1,2,2,3,3 so the probability of producing a 1, 2 or 3 is  $1/3$ . The following experiment is performed. The die is rolled repeatedly until the die produces a 1 and the number of rolls required to produce the first 1 is recorded. Then the experiment is repeated (i.e. the die is rolled repeatedly until the first 1 is produced and the number of rolls required to produce the first 1 is recorded.) What is the probability that the two experiments produce the same result.

- A.  $1/10$  B.  $1/9$  C.  $1/7$  D.  $1/5$  E.  $1/4$  F.  $2/7$  G.  $1/3$  H.  $2/5$   
 ##

4. There are twelve six sided dice. Three of the dice have all ones, six dice have four ones and the remaining sides are blank and three of the dice have two ones and the remaining sides are blank. One of the twelve dice is selected at random and tossed twice. The die produces a one on each toss. What is the probability it is one of the dice with all ones?

- A.  $1/9$  B.  $1/6$  C.  $2/9$  D.  $1/3$  E.  $4/9$  F.  $1/2$  G.  $5/9$  H.  $2/3$   
 ##

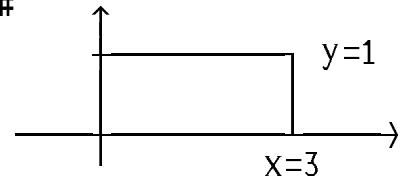
5. Four fair dice A, B, C and D numbered 1,1,2,2,3,3 (so the probability of producing any of the numbers 1-3 is  $1/3$ ) are rolled. Suppose all four dice are rolled. Let  $X$  be the total number of spots showing. Compute the variance of  $X$ . You are given that the die are independent. (Hint. the die are independent.) The variance of  $X$  =

- A. 2 B.  $17/3$  C.  $35/4$  D.  $8/3$  E. 4 F.  $19/4$  G. 3 H.  $11/2$   
 ##

6. The joint probability distribution function for  $X$  and  $Y$  where  $0 \leq X \leq 3$  and  $0 \leq Y \leq 1$  is  $f(x,y) = \frac{4}{9}xy$ . Compute the probability that  $X > Y$ .

A.  $1/2$  B.  $5/9$  C.  $2/3$  D.  $3/4$  E.  $5/6$  F.  $8/9$  G.  $17/18$  H.  $15/16$

##



7. The number of clicks of a Geiger counter in any given time interval is a Poisson process. The average number of clicks per minute is 12. What is the probability there will be three or more clicks in a ten second interval?

A.  $e^{-4}$  B.  $1-13e^{-4}$  C.  $1/e$  D.  $1-(7/3)e^{-3}$  E.  $1-7e^{-4}$  F.  $1-4e^{-2}$  G.  $2/3$  H.  $1-5e^{-2}$

##

8. There are 5 balls in a box and 3 are red and 2 are green. Balls are drawn out one at a time without replacement until a red ball is drawn. What is the expected number of balls that will be drawn until a red ball is drawn.

A.  $4/3$  B.  $3/2$  C.  $11/7$  D.  $5/3$  E.  $13/7$  F.  $23/12$  G.  $2$  H.  $9/4$

##

9. How many ways can you arrange the letters AABBCCCC?

A. 48 B. 65 C. 120 D. 240 E. 350 F. 420 G. 480 H. 520

##

10. Suppose  $X$  is an exponentially distributed random variable with mean two seconds and  $Y$  is an exponentially distributed random variable with mean four seconds. (The probability density function for an exponentially distributed random variable with mean  $m$  is  $f(t) = (1/m)e^{-t/m}$  for  $t \geq 0$ ) Given that the random variables  $X$  and  $Y$  are independent compute the probability that  $X$  occurs after  $Y$  (i.e.  $\text{Prob}(X > Y)$ ). (To get credit you must set up and evaluate the double integral.)

A. 0 B.  $1/5$  C.  $1/3$  D.  $2/5$  E.  $1/2$  F.  $3/5$  G.  $2/3$  H.  $4/5$

##

11. Suppose a player starts with a fortune of 8 dollars. A fair coin is tossed three times. If the coin comes up heads the player's fortune is doubled and if the coin comes up tails the player's fortune is halved. What is the expected fortune of the player at the end of the game after three coin tosses.

A.  $\frac{95}{8}$  B.  $\frac{105}{8}$  C.  $\frac{115}{7}$  D.  $\frac{125}{8}$  E. 10 F. 8 G. 11 H. 14  
##

12. Find the best least squares fit to the four points  $(x,y) = (0,1), (1,2), (1,4)$  and  $(2,5)$ .

A.  $y = -x + 6$  B.  $y = x + 2$  C.  $y = 2x + 3$  D.  $y = 2x + 1$  E.  $y = -2x + 2$   
##  
F.  $y = -2x$  G.  $y = -2x + 4$  H.  $y = -3x + 4$

13. An unfair coin has a  $\frac{3}{7}$  probability of producing a heads and a  $\frac{4}{7}$  probability of producing a tails. The coin is flipped  $147 = (3 \times 7 \times 7)$  times. Using the table of the standard normal distribution provided estimate the probability that there are more heads than tails (i.e. 74 or more heads)? Circle the closest answer. (No credit will be given unless you show how you calculated the result, what you looked up and what you did with it. (e.g.  $\Phi(1.5) - \Phi(0.5) = 0.9332 - 0.6915 \approx .24$ ))

A. 17% B. 12% C. 8% D. 5% E. 3% F. 1% G. 0.1% H. 0.05%  
##

$1/7 = .1426$
$2/7 = .2857$
$3/7 = .4286$
$4/7 = .5714$
$5/7 = .7143$
$6/7 = .8571$
$1/6 = .1667$
$1/3 = .3333$
$2/3 = .6667$
$5/6 = .8333$