

1 Differential Geometry

(HG)1. Is there a closed curve γ in the plane such that for any point $p \in \mathbb{R}^2$, p can be connected to γ by a tangent line?

(WZ)2. Let M_g be a closed orientable surface of genus g with $sec = -1$.

a. What g are possible?

b. What are the cut locus and conjugate locus of $M_g \times S^1$, where S^1 has the usual round metric of radius 1.

i. Why are geodesics on $M_g \times S^1$ simply geodesics in each factor?

ii. Why are Jacobi fields on $M_g \times S^1$ simply Jacobi fields on each factor?

(HG)3a. Let M be a 3-manifold, with $p \in M$ any point. Suppose you know $Ric_p(X)$ for all $X \in T_pM$. Let P be any 2-plane in T_pM . Can you determine $K(P)$ (the sectional curvature of P)?

3b. Now suppose all of the above hypothesis, except now M is a 4-manifold. Can you determine $K(P)$?

(HG)4. If M is a connected surface, must the cut locus be connected?

(WZ)5a. Let M be a compact manifold. Show that in each nontrivial free homotopy class of loops, there is a closed geodesic.

(HG)5b. Give an example where the closed geodesic isn't simple.

(WZ)6a. Can the Christoffel symbols Γ_{ij}^k be constant?

(WZ)6b. Show that around any point $p \in M$ there are coordinates such that $\Gamma_{ij}^k(p) = 0$.

(WZ)6c. Let γ be a geodesic. Show there are coordinates around γ such that $\Gamma_{ij}^k(\gamma(t)) = 0$ for all t .

2 Set Theory/Logic

1. How many countable models of Peano arithmetic are there?

2. Show the following 2 statements are equivalent:

a. There is a nontrivial elementary embedding $j : V \rightarrow M$

b. There is a measurable cardinal κ