

Orals Examination Syllabus

Major Area: Algebraic Topology

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1. Fundamental Group and Covering Spaces (Hatcher)
 - (a) The Seifert-Van Kampen theorem
 - (b) Lifting properties and Deck Transformations
2. Homology and Cohomology (Hatcher, Bredon, Bott & Tu)
 - (a) Singular and Cellular homology
 - (b) Eilenberg-Steenrod axioms
 - (c) Mayer-Vietoris sequence
 - (d) The Universal Coefficient Theorems for Homology and Cohomology
 - (e) Cross product and the Künneth formula
 - (f) Cup and cap products
 - (g) Poincaré and Alexander duality
 - (h) De Rham cohomology and differential forms
3. Homotopy Theory (Hatcher, Bott & Tu)
 - (a) Higher homotopy groups and long exact sequences in homotopy, Fibrations
 - (b) Whitehead's Theorem
 - (c) Excision for homotopy groups, Freudenthal Suspension Theorem
 - (d) Hurewicz's Theorem
 - (e) Brown Representability
4. Serre Spectral Sequence and Applications (Hatcher SSAT)
5. Bundle Theory and Characteristic classes (Milnor & Stasheff through §16, Bott & Tu)
 - (a) Euler, Steifel-Whitney, Chern and Pontrijagin classes
 - (b) Classifying spaces
 - (c) Thom isomorphism and the Gysin sequence
6. Morse Theory (Nicolaescu's Invitation to Morse Theory, Chapters 1,2)
 - (a) Morse Functions
 - i. The Local Structure of Morse Functions
 - ii. Existence of Morse Functions
 - (b) The Topology of Morse Functions
 - i. Surgery, Handle Attachment, and Cobordisms
 - ii. The Topology of Sublevel Sets
 - iii. Morse Inequalities
 - iv. Morse-Smale Dynamics
 - v. Morse-Bott Functions
 - vi. Min-Max Theory