

Orals Examination Syllabus

Minor: PDEs

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1. Fundamentals of Real Analysis, emphasis on Banach and Hilbert Space Theory; *par exemplar*
 - (a) Bounded if and only if Continuous
 - (b) Open Mapping Theorem
 - (c) Hahn-Banach Theorem
 - (d) Riesz Representation Theorem
 - (e) Contraction Mapping Theorem (Banach Fixed Point Theorem)
2. Partial Differential Equations
 - (a) Laplace's and Poisson's Equation
 - i. Fundamental Radial Solutions
 - ii. Mean-Value Property, Max and Min Principles
 - iii. Representation Formulae, Green's Functions for Ball and Half Space
 - iv. Perron's Method, Properties of Sub/Super-Harmonic Functions
 - v. Boundary Regularity, Barriers
 - (b) Weak Solutions and Sobolev Spaces
 - i. Hölder Spaces
 - ii. Sobolev Embedding Theorems: Rellich Theorem, Kondrachev Compactness
 - iii. Weak Derivatives, Weak Solutions, *etcetera*
 - (c) General Variable Elliptic PDE through Functional Analysis
 - i. Lax-Milgram Theorem
 - ii. Fredholm Alternative
 - iii. Application to Hodge Decomposition Theorem
 - (d) General First Order Nonlinear PDE
 - i. Method of Characteristics: ODE, Boundary Conditions, Application
 - ii. General Conservation Laws: Formation of Shocks, Rankine-Hugoniot
 - iii. Burger's Equation: Hopf's Method, Uniqueness of Solutions, Entropy Criterion
 - iv. Hamilton-Jacobi Equations: Calcululus of Variations, Legendre Transform, Hopf-Lax
3. **References:**
 - (a) Evans "Partial Differential Equations" Chapters 2.2, 3, 5, 6 and Appendices
 - (b) Gilbarg and Trudinger "Elliptic Partial Differential Equations of Second Order" Chapters 2, 3, 4, 5, 7, 8
 - (c) Hörmander "Lectures on Hyperbolic Non-linear Differential Equation" Ch. 2