HOMEWORK 1 - MATH 170, SUMMER SESSION I (2012)

Instructions: Here is our very first homework for the course. As I had mentioned, it is due in class on Tuesday, May 29 (the day after Memorial Day). The course policy forbids acceptance of late homeworks.

• All the questions are compulsory.
• Do box or underline your final answer.
• For all the questions, SHOW YOUR STEPS. I'm afraid I cannot put a deserving grade to a solution which only has the final answer but provides no explanation as to how one arrived at it (even if the final answer is indeed right). If you feel there are no "mathematical steps" to obtain the answer, explain in words. Believe me, you can reason a lot of Math in just plain English!
• Do not use theorems or results that have not been taught in the lectures or are not a part of the course. Trust me, each of the problems below rely on what you already know... just leaf through the notes, remember the important results we did in class and the techniques we used to solve examples, and try to apply them here.
• Please do not use calculators. If you do, then firstly, you will not be able to show your work, and secondly, you would become increasingly dependent on them and would find it hard to function without one during the quiz or exam.
• As we had discussed before, working together to solve Math is something I encourage, but plagiarism is something I'm completely against, and so in the university. Make sure you maintain the right balance when working with people. Penn even has a website (http://www.upenn.edu/academicintegrity/) that helps to underline what is and is not a violation of academic integrity.

(1) In class, we had used Pigeonhole Principle to show that if we choose any 6 numbers from $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, at least two of those numbers must add up to 10. Along the same lines,

(a) can you show the more general statement that for any n, if we choose any $n+1$ numbers from $\{1, 2, 3, ..., 2n-1\}$, at least two of those chosen numbers add up to $2n$?

(b) What if we were to choose n numbers from $\{1, 2, 3, ..., 2n-1\}$? Does the same conclusion hold? That is, will there still be at least two numbers from the chosen ones that add up to $2n$? If you think yes, show with good arguments.

If you think no, give an example where it does not hold.

(2) (Exercise 14, Page 60) You have 10 pairs of socks, 5 black pairs and 5 blue pairs, but they are not paired up. Instead, they are all mixed up in a drawer. It's early in the morning and you don't want to turn on the lights in your dark room. How many socks must you pull out to guarantee that you have a pair of one color? How many must you pull out to have two good pairs (each pair is the same color)? How many must you pull out to be certain that you have a pair of black socks?

Hint: In each of the three questions asked, first guess a number for which the statement may be true using your intuition, and then show that it is indeed true using Pigeonhole Principle.

(3) This is the question that Leonardo of Pisa (or Fibonacci) asked himself, and this came up with the famous Fibonacci numbers.

Suppose we have a pair of baby rabbits: one male and one female. Let us assume that the rabbits cannot reproduce until they are one month old, and that they have a one-month gestation period. Once they start reproducing, they produce a pair of bunnies each month (one of each sex). Let us assume here that no pair ever dies.

(a) Fill in (out?) the following table:

Time in Months	Number of Pairs
0 (Start)	1
1	1
2	
3	
4	
5	
6	
7	
8	

(b) Guess a pattern that the quantity "the number of pairs of bunnies at the end of a month" follows. Explain clearly and in words why you predict this pattern (your explanation should be based on the information you have about how quickly the baby bunnies grow and what the gestation period is... not on the table above).

(4) (Exercise 16, Page 73) Express each of the following natural numbers as a sum of distinct, nonconsecutive Fibonacci numbers: 52, 143, 13, 88.

(5) We showed in class that the square of any natural number is either divisible by 4 or leaves a remainder of 1 when divided by 4. Show now that the square of any natural number is either divisible by 3 or leaves a remainder of 1 when divided by 3.

(6) True or False: If a and b are natural numbers with a divides b and b divides a, then $a = b$.

If you think this statement is true, show this mathematically. If you think it is false, then give an example where this does not hold.

(7) (a) Write down the first 8 twin prime pairs. Also, next to each pair, write down its sum. (For example, the first twin prime pair is (3,5) and their sum is 3+5=8, and so on).

(b) We showed in class that if p and $p+2$ are twin primes, with $p > 3$, then 3 divides $p+1$. Using this fact and other knowledge you have about primes and integers in general, show also that if p and $p+2$ are twin primes, with $p > 3$, then their sum is divisible by 12.

(8) Compute the following:

(a) 5^3 + (148-71)×43 $\mod 16$

(b) $6^{102} \mod 11$

(9) (Exercise 2, Page 106) Today is Saturday. What day of the week will it be in 3724 days? What day of the week will it be in 365 days?