HOMEWORK 2 - MATH 170, SUMMER SESSION I (2012)

INSTRUCTIONS: This homework assignment is due in class on Tuesday, June 5th. The main instructions are the same as before, with the addition of a new one - please staple your homework before handing it in.

(1) (Exercise 11, Page 107) Which of the following is the correct UPC for Progresso minestrone soup? Show why the other numbers are not valid UPC's.
0 41196 01012 1
0 52010 00121 2
0 05055 00505 3
Note: UPC stands for Universal Product Code, which is basically the product bar codes we had been discussing in lectures.

(2) (Exercise 13, Page 107) The following is the UPC for Hellman's 8 oz. Real Mayonnaise. Find the missing digit.
0 48001 26 ■ 04 2

(3) A bank identification number is a 9 digit number that occurs in the lower left hand corner of bank checks. Let the digits be $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9$. For a 9 digit number to be a valid bank identification number, it must satisfy

$$7n_1 + 3n_2 + 9n_3 + 7n_4 + 3n_5 + 9n_6 + 7n_7 + 3n_8 + 9n_9 \equiv 0 \mod 10.$$

(Exercise 21, Page 108) Determine the check digits (i.e. last digits) of the following bank codes:
3 1 0 6 1 4 8 3 ■
0 2 5 7 1 1 0 8 ■

(4) Show that the following statements are true:
(a) $5^{47} \equiv 6 \times 17 \mod 23$.

(b) $4^2 \times 33 \equiv 2^{51} \mod 13$.

(5) Carefully look at the following mathematical reasoning. Write down what is wrong with it.

I believe that $68^{32} \equiv 1 \mod 17$. This is because, Fermat's theorem says that if p is prime, $a^{p-1} \equiv 1 \mod p$, and since 17 is a prime with $17 - 1 = 16$, we must have, $68^{16} \equiv 1 \mod 17$. Taking a power of 2 on both sides of the congruence, we get, $68^{32} \equiv 1 \mod 17$.

After you have answered what is the mistake above, write down the correct number between 0 and 16 that is $68^{32} \mod 17$.

(6) Prove the following statements using the principle of induction:

(a) $1 + 3 + 5 + ... + (2n - 1) = n^2$, for all natural numbers n.
(b) $3^{2n-1} + 2^{n+1}$ is divisible by 7, for all natural numbers n.
(c) Let F_n denote the nth Fibonacci number (so, $F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3$ and so on). Show that

$$F_1^2 + F_2^2 + ... + F_n^2 = F_n \cdot F_{n+1}, \text{ for all natural numbers } n.$$

(7) Prove the following statements using the method of contradiction:
(a) The negative of an irrational number is irrational.
(b) There are no even primes that are bigger than 2.
(c) $\sqrt{6}$ is irrational.
(d) $\sqrt{2} + \sqrt{3}$ is irrational.

(8) Let A and B be two sets. Draw Venn diagrams and shade the regions for the following set expressions:
(a) $(A - B)^c$.
(b) $(A \triangle B) - A$.

(9) Prove the following statements (not using Venn diagrams):
(a) $A \cap (B \cap C) = (A \cap B) \cap C$.
(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
(c) $(A \cap B)^c = A^c \cup B^c$.