

HOMEWORK 3 - MATH 170, SUMMER SESSION I (2012)

This homework assignment is due in class on Tuesday, June 12, 2012.

- (1) Let \mathcal{U} be a universal set and $A \subset \mathcal{U}$. Using rules that you have learnt in class, simplify the following set theoretic expressions:
 - (a) $((A^c \cup A) - A) \cap A$.
 - (b) $((A \Delta A^c) \cap (A^c - \mathcal{U}))$.

- (2) Suppose you have a tetrahedron (which is a four sided symmetric object), with each of its four sides numbered 1, 2, 3, 4, and a decahedron (which is a ten sided symmetric object), and you toss both of them and note down the number that comes up on each.
 - (a) Write down the universal set of this experiment.
 - (b) Express the event of getting a sum of 12 when you add up the two numbers in the tosses.
 - (c) Write down the probability of that event.
 - (d) Express the event of getting a product of 12 when you add up the two numbers in the tosses.
 - (e) Write down the probability of that event.

- (3) A friend of yours has a coin that shows heads or tails. He tosses it once, notes down the outcome, then tosses it the second time, notes down the outcome, then the third time, and so on. At some point, you ask him when he intends to stop, to which he replies that he would stop when the coin gives him a head as its outcome for the third time.
 - (a) Find the probability that he stops tossing the coin after 5 tosses.
 - (b) Find the probability that he does not stop in 4 tosses.

- (c) Can he stop after just two tosses?
- (4) Given a big box with 18 green and 12 red ball, say you draw three balls in the following manner. After each draw, you look at the color of the ball and replace it back in the box, after which you make the next draw.
- (a) What is the probability that all the balls drawn are green?
- (b) What is the probability that at least one of the balls is red?
- (c) What is the probability that exactly one ball is red?
- (d) If I were to change the question, wherein, you draw the three balls one by one without replacing any of them, in that case, how would you answer parts (a), (b) and (c) of this problem?
- (5) You now have two big boxes... Box 1 has 10 white and 10 black balls, and Box 2 has 14 white and 18 black balls. You choose a ball at random from Box 1, note its color and then drop it into Box 2. After that you choose a ball are random from Box 2.
- (a) What is the probability that both the balls turn out to be white?
- (b) What is the probability that both the balls turn out to be black?
- (c) What is the probability that the second ball is black? Is this higher than the probability that the second ball is white?