HOMEWORK 4 - MATH 170, SUMMER SESSION I (2012)

This homework assignment is due in class on Tuesday, June 19, 2012.

(1) Let a and b be any two fixed real numbers. Show using induction that

$$(a+b)^n = \binom{n}{0}b^n + \binom{n}{1}a \cdot b^{n-1} + \binom{n}{2}a^2 \cdot b^{n-2} + \cdots + \binom{n}{n}a^n$$
$$= \sum_{k=0}^{n} \binom{n}{k}a^k b^{n-k},$$

for all natural numbers n.

(2) What is the number of permutations (or rearrangements) that can be formed from all the letters of each word:

   (a) BASKET
   (b) QUEUE
   (c) COMMITTEE
   (d) PROPOSITION
   (e) compute this for your own (full) name

(3) A debating team consists of 6 boys and 6 girls. Find the number of ways they can sit in a row where,
   (a) there are no restrictions.
   (b) the boys and girls are each to sit together.
   (c) just the girls are to sit together.

(4) A box has 10 blue socks and 8 white socks.

   (a) What is the total number of ways two socks can be drawn from this box?
   (b) What is the number of ways two socks can be drawn such that they are of different colors?
   (c) If the two socks were drawn randomly, what then is the probability that they turn out to be of different colors?
   (d) What is the number of ways two socks can be drawn such that they are of the same color?

    (e) If the two socks were drawn randomly, what then is the probability that they turn out to be of the same color?

(5) A password consists of two letters of the alphabet followed by three digits chosen from 0 to 9.

    (a) If repeats are allowed, how many different possible passwords are there?
    (b) If repeats are not allowed, how many different possible passwords are there?
    (c) If repeats are allowed among the digits but not among the letters, how many different possible passwords are there?

(6) You are playing poker with your friends and are dealt a hand (five cards) from a regular deck.

    (a) How many possible combinations of five cards are there?
    (b) What is the probability that you will get a straight flush (that is, all of your five cards will be from the same suit and of sequential rank (i.e. will have consecutive numbers on them)?
    (c) What is the probability that you will get a flush (that is, all the cards will be of the same suit, but NOT in a sequence)?
    (d) What is the probability that you will get four of a kind (that is, four out of the five cards will be of the same rank)?
    (e) Explain, based on your answers to parts (b), (c) and (d) as to why, in a game of poker, a straight flush has a higher value than a four of a kind, which in turn has a higher value than a flush.

    Hint: You might want to use a calculator for parts (b), (c) and (d) to get the exact final answer, which will allow you to answer part (e).

(7) A billiard ball is drawn at random from a box containing 15 billiard balls numbered 1 to 15. The number on the ball drawn (call is n) is recorded.

    (a) Find the probability that n is greater than 8.
    (b) If given that n is even, find the probability that n is greater than 8.
    (c) If given that n is a prime, find the probability that n is greater than 8.

(8) In a country club, 65 percent of the members play tennis, 40 percent play golf, and 20 percent play both tennis and golf. A member is chosen at random. Find the probability that the chosen member
    (a) plays tennis or golf.
    (b) plays neither tennis nor golf.
    (c) plays golf, given that he or she plays tennis.
    (d) plays tennis, given that he or she plays golf.