HOMEWORK 3 - DUE TUES. FEB. 22

1. a) Prove that n^5 has the same last digit as n.
b) Prove that if $gcd(n, 100) = 1$, then n^{41} has the same last two digits as n.

2.) Show that if p is prime, then

$$1 + 1^{p-1} + 2^{p-1} + \cdots + (p-1)^{p-1} = 0(\mathrm{mod}p)$$

3.) Prove that a number is divisible by 9 iff the sum of its digits is divisible by 9. Find a similar criterion for 11.

4.) Prove that if $n_1, \cdots, n_k$ are positive numbers such that $gcd(n_i, n_j) = 1$ for $i \neq j$, then

$$\mathbb{Z}/n_1 n_2 \cdots n_k\mathbb{Z} \cong \mathbb{Z}/n_1\mathbb{Z} \times \cdots \times \mathbb{Z}/n_k\mathbb{Z}$$