MATH 371 MIDTERM - DUE MONDAY MARCH 24 IN CLASS

1. Prove that the rings $\mathbb{C}[x,y]/(y-x^2)$ and $\mathbb{C}[t]$ are isomorphic.

2. Prove that the polynomial x^3-3x-1 is irreducible in $\mathbb{Z}[x]$.

3. Let a and b be to relatively prime positive integers. Prove that for $N > ab$, the equation $ax+by=N$ can be solved with both x and y positive integers.

4. Let F be a field. Describe all the ideals of the ring $F[x]/(p(x))$, where $p(x) \in F[x]$. (Hint: use the factorization of $p(x)$).

5. Let R be a non-zero ring such that every finitely generated R–module is free. Prove that R is a field.

6. a) Classify all finitely generated modules over the ring $\mathbb{C}[x]/(x^2)$

b) Classify all finitely generated modules over the ring $\mathbb{Z}/n\mathbb{Z}$.

7. (Jordan form over $\mathbb{R}$) Let V be a finite-dimensional vector space over $\mathbb{R}$, and $T: V \mapsto V$ a linear tranformation. Show that there exists a basis of V such that the matrix of T in that basis is made up blocks of the following two types:
i)

$$\begin{bmatrix} \lambda & 0 & \cdots & 0 \\ 1 & \lambda & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda \end{bmatrix}$$

where $\lambda \in \mathbb{R}$, and
ii)

$$\begin{bmatrix} D & 0 & \cdots & 0 \\ I_2 & D & \cdots & 0 \\ 0 & I_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & D \end{bmatrix}$$

where $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ $a, b \in \mathbb{R}$