PRACTICE QUESTIONS FOR MIDTERM 2

The second midterm will cover linear transformations, changes of basis, determinants, eigenvectors/eigenvalues, and diagonalization.

1. a) Write down the matrix representing a reflection in the line $y = x$ with respect to the standard basis
b) Do the same with projection onto the line $y = x$.

2.) Write down the equation of the hyperbola with asymptotes $y = x$, $y = -x$, passing through the point $(2, 0)$.

3.) If the matrix of a linear transformation T with respect to a basis $\{v_1, v_2\}$ is

$$A = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix},$$

what is the matrix with respect to the basis $\{w_1, w_2\}$ where $w_1 = 2v_1 + v_2$, $w_2 = -v_1 + v_2$?

4.) compute A^{20} where A is the matrix

$$\begin{pmatrix} 1 & -4 \\ 1 & 1 \end{pmatrix}$$

5.) Determine whether the statements are true or false. If true, say why, if false, give a counterexample.

(1) The transformation on the set of continuous functions defined by $T(f) = f(1)$ is a linear transformation
(2) The transformation defined on the set of differentiable functions given by $T(f) = df/dx + 1$ is a linear transformation.
(3) It is impossible for a linear transformation $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ to transform a parallelogram to a line segment.
(4) There exists a linear transformation $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ such that $T(1, 1) = (4, 5)$ and $T(2, 2) = (5, 6)$.
(5) If $T : V \mapsto W$ is a linear transformation between the vector spaces V and W, and v_1, v_2, v_3 is a set of three linearly dependent vectors, then the images $T(v_1), T(v_2), T(v_3)$ are also linearly dependent.
(6) If A, B are $n \times n$ matrices, then $det(A + B) = det(A) + det(B)$.
(7) Let A be an $n \times n$ matrix such that $Ax = b$ fails to have a solution for some b, then $det(A) = 0$

(8) Let A be a 3×3 matrix such that $A \times (1, 2, 1)^T = 0$, then $det(A) = 0$.

(9) Let C be a $(n + m) \times (n + m)$ matrix which has the form

$$\begin{pmatrix} A & D \\ 0 & B \end{pmatrix}$$

where A is $n \times n$, B is $m \times m$, and D is $n \times m$. Then $det(C) = det(A) \times det(B)$.

(10) Eigenvectors corresponding to distinct eigenvalues are linearly independent.

(11) If A is an $n \times n$ matrix such that $det(A) = 0$, then $\lambda = 0$ is an eigenvalue of A.

(12) The sum of two eigenvectors is an eigenvector.