PRACTICE QUESTIONS FOR MIDTERM 1

1. Let

$$A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 1 & 1 & 2 & 1 \\ 2 & 3 & 2 & 2 \\ 0 & 1 & -2 & 0 \end{pmatrix}$$

a) Find the rank of A
b) Find the dimensions of and bases for *rowsp(A)*, *colsp(A)*, *nullsp(A)*
c) Determine whether the vector $(1, 0, -2, 3) \in$ *rowsp(A)*
d) Extend the basis for $colsp(A)$ to a basis for $\mathbb{R}^4$.

2. Determine whether

$$B = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$$

is non-singular. If so, find B^{-1}.

3. Find a basis for the smallest subspace of $\mathbb{R}^6$ containing the two subspaces $W_1 = span\{(1, 2, 3, 0, 3, 2), (0, 0, 1, 2, 1, 0)\}$ and $W_2 = span\{(1, 2, 2, -2, 2, 2), (0, 0, 1, 2, 1, 1)\}$.

4.) Determine whether the statements are true or false. If true, say why, if false, give a counterexample.

(1) Let A be a 3×5 matrix such that $(1, 1, 1, 1, 1)$, $(0, 1, 1, 1, 1)$, $(0, 0, 1, 1, 1)$ are in *nullsp(A)*. The rows of A are linearly independent.
(2) With A as in 1,the equation $Ax = B$ has a solution for every $B \in \mathbb{R}^3$.
(3) For A as in 1, the solution to $Ax = B$ when it exists is unique.
(4) A 5×27 matrix can have 6 linearly independent columns.
(5) Suppose that A is an $n \times n$ matrix and $B \in \mathbb{R}^n$ a vector such that $Ax = B$ has infinitely many solutions. Then the columns of A are linearly dependent.
(6) Suppose A is a 3×7 matrix such that $Ax = B$ is solvable for every $B \in \mathbb{R}^3$. Then A has 3 linearly independent rows.
(7) Suppose that A is a matrix such that *colsp(A)* and *nullsp(A)* are both 2-dimensional. Then A must be a 4×4 matrix.

(8) Suppose that A is an $n \times n$ matrix such that $Ax = B$ is solvable for every $B \in \mathbb{R}^n$. Then for every $n \times n$ matrix C, there exists a matrix D such that $AD = C$.
(9) The nullspace of a 3×4 matrix cannot consist only of the zero vector.
(10) The nullspace of a 4×3 matrix cannot consist only of the zero vector.
(11) Let $S = \{v_1, v_2, v_3, v_4, v_5\}$ be a set of vectors in $\mathbb{R}^4$ that spans $\mathbb{R}^4$. Every subset of four vectors of S spans $\mathbb{R}^4$.
(12) The set of all vector of the form $(1, x, y) \subset \mathbb{R}^3$, where $x, y \in \mathbb{R}$, is a subspace of $\mathbb{R}^3$.
(13) Let u, v, w be elements of a vector space V, and let $a = u+v, b = w$. Then $span\{a, b\}$ is contained in, but not necessarily equal to $span\{u, v, w\}$.
(14) The set of all vectors of the form $(x, x^2) \in \mathbb{R}^2$, as x ranges over the real numbers, form a subspace of $\mathbb{R}^2$.
(15) All vector spaces are finite-dimensional.