SOLUTIONS FOR HOMEWORK 4

1. a) Identifying A, B, C, D with $1, 2, 3, 4$ resp.,

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

b) Compute M^3 and take the $(1, 3)$–entry.

2.) a) You need to check that

$$T(v + w) = T(v) + T(w)$$
$$T(cv) = cT(v)$$

To check the first part, let $v = (x_1, x_2, x_3, x_4)$ and $w = (y_1, y_2, y_3, y_4)$. Then

$$\begin{aligned} T(v + w) &= T(x_1 + y_1, x_2 + y_2, x_3 + y_3, x_4 + y_4) \\ &= ((x_2 + y_2) + 3(x_4 + y_4) - (x_1 + y_1), (x_2 + y_2) - (x_4 + y_4)) \\ &= (x_2 + 3x_4 - x_1, x_2 - x_4) + (y_2 + 3y_4 - y_1, y_2 - y_4) \\ &= T(v) + T(w) \end{aligned}$$

The second part is checked similarly.

b)

$$A_T = \begin{pmatrix} -1 & 1 & 0 & 3 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

3.) By using suitable linear combinations of vectors, it follows that

$$T(1, 0, 0) = (3, -3, 3)$$
$$T(0, 1, 0) = (-2, 2, 0)$$
$$T(0, 0, 1) = (8, -6, 1)$$

so

$$\begin{pmatrix} 3 & -2 & 8 \\ -3 & 2 & -6 \\ 3 & 0 & 1 \end{pmatrix}$$

4.) Rotating the parabola by $\pi/4$ yields a parabola with axis of symmetry the y'-axis, and therefore has an equation of the form $y' = a(x')^2 + b$ (we call the original axes x, y and the rotated ones x', y').

$(1, 1)$ rotates to $(0, \sqrt{2})$ so $b = \sqrt{2}$. $(1, 2)$ rotates to $(-\frac{1}{\sqrt{2}}, \frac{3}{\sqrt{2}})$, which implies that $a = 2\sqrt{2}$. The equation is therefore

$$(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}) = 2\sqrt{2}(\frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}})^2 + \sqrt{2}$$

This expression can be further simplified.