SOLUTIONS FOR HOMEWORK 5

1. The matrix with respect to the f–basis is $A_{T,\{e_1,e_2,e_3\}} = Q^{-1}A_{T,\{e_1,e_2,e_3\}}Q$, where

$$Q = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 2 & 2 \\ 0 & 1 & -2 \end{pmatrix}$$

2.) A natural basis for the space of polynomials of degree ≤ 2 is given by $e_1 = 1, e_2 = x, e_3 = x^2$. Letting $T = d/dx$, the matrix of T with respect to the e–basis is

$$A_{T,\{e_1,e_2,e_3\}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

Let $f_1 = 1 + x$, $f_2 = 2x$, $f_3 = 3x^2 - 1$. We have, as above $A_{T,\{e_1,e_2,e_3\}} = Q^{-1}A_{T,\{e_1,e_2,e_3\}}Q$, where

$$Q = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

3.) This problem was done in class.

2.6.7) i)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

ii)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

iii)

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

iv)

$$\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{pmatrix} \begin{pmatrix}0 & 0 & -1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{pmatrix} \begin{pmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}$$

v)

$$\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{pmatrix} \begin{pmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{pmatrix} \begin{pmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}$$

2.26) a) True - take the projection onto the orthogonal complement.
b) True - if A is $m \times n$, $nullsp(A) \in \mathbb{R}^n$, and $nullsp(A^T) \in \mathbb{R}^m$. Thus $m = n$.
c) False - $m(x_1 + x_2) + b \neq (mx_1 + b) + (mx_2 + b)$ unless $b = 0$.

4.2.13) If every row adds to 0, the columns of A add to the zero vector, which implies that the columns are linearly dependent, and $det(A) = 0$. If every row adds to 1, the columns add to the vector $v = (1, 1, \cdots, 1)^T$. This can be rewritten $Av = v$, or $(A - I)v = 0$, which implies that $det(A - I) = 0$. For the last part, consider the 2×2 matrix all of whose entries are $1/2$.