

## PRACTICE MIDTERM II PROBLEMS

1. For which  $n$  does  $\phi(n)|n$  ?

2. Suppose that  $f$  is a multiplicative function. Show that

$$\sum_{d|n} \mu(d)f(d) = (1 - f(p_1))(1 - f(p_2)) \cdots (1 - f(p_k))$$

where  $n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$  is the prime power factorization of  $n$ .

3. Evaluate the following Legendre symbols:

a)  $\left(\frac{71}{547}\right)$

b)  $\left(\frac{1776}{1511}\right)$

4. Evaluate the following Jacobi symbols:

a)  $\left(\frac{210}{95}\right)$

b)  $\left(\frac{73}{325}\right)$

5. Let  $m = a^n - 1$  where  $a$  and  $n$  are positive integers. Show that  $n|\phi(m)$ .

6. What are the primitive roots  $\text{mod } 7^3$  ?

7. Show that if  $b$  is a positive integer not divisible by the prime  $p$ , then

$$\left(\frac{b}{p}\right) + \left(\frac{2b}{p}\right) + \left(\frac{3b}{p}\right) + \cdots + \left(\frac{(p-1)b}{p}\right) = 0$$