

Combinatorics Syllabus - Orals Major Topic

Part I - Enumerative Combinatorics

Unlabelled Combinatorial Classes (Ch. 1, [Fla06])

- Admissible Constructions and Specifications
(disjoint union, cartesian product, sequence, cycle, multiset, powerset)
- Integer Compositions, Partitions and Related Constructions
(Durfee squares, stack polyominoes)
- Words and Regular Languages
(run statistics, patterns, autocorrelation)
- Tree Structures
(Lagrange inversion theorem, triangulations)
- Additional Constructions
(restricted constructions, pointing, substitution, implicit specification)

Labelled Combinatorial Classes (Ch. 2, [Fla06])

- Admissible Labelled Constructions
(labelled product, sequence, set, cycle)
- Surjections, Partitions and Words
(Stirling partition numbers, arrangements, birthday paradox and coupon collector problems)
- Alignments, Permutations and Related Structures
(Stirling cycle numbers, involutions, derangements)
- Labelled Trees, Mappings and Graphs
(Cayley and Catalan trees, functional graphs)
- Additional Constructions
(pointing, substitution, implicit specification, boxed and max-boxed products)

Combinatorics on Words (Ch. 7, [Lot04])

- Correlation and Autocorrelation Polynomials
- Tries (independent and suffix, internal versus external profiles)

Multivariate Generating Functions (Ch. 3, [Fla06])

- Bivariate Generating Functions and Probability Distributions
(horizontal and vertical GFs, parameters, power and factorial moments, expectation and standard deviation, cumulative GFs, Markov-Chebyshev inequalities, Poisson Law)
- Inherited Parameters and Ordinary Multivariate Generating Functions
- Inherited Parameters and Exponential Multivariate Generating Functions
- Recursive Parameters
(linear transformations of parameters)
- Complete Generating Functions and Discrete Models
(Bernoulli trials, word and tree models, Galton-Watson processes)

- Additional Constructions
(pointing, substitution, implicit specification, constrained integer compositions, inclusion-exclusion)
- Extremal Parameters

Part II - Asymptotics

Background on Analysis and Generating Functions

- Asymptotic Notation (Chapter 1, [Pem06])
- Ring of Germs of Analytic Functions (Chapter 2, [Pem06])
- Kernel Method (Chapter 2, [Pem06]; Chapter 7, [Fla06])
- Transfer Matrices (Chapter 2, [Pem06]; Chapter 5, [Fla06])
- Generating Function Classes (D-finite, P-recursive, algebraic, etc.; Chapter 2, [Pem06])

Univariate Methods

- Circle Methods (Chapter 3, [Pem06]; Chapter 6, [Fla06])
Tauberian Theory, Darboux's Theorem and Transfer Theorems
- Saddle Point Methods (Chapter 3, [Pem06]; Chapter 8, [Fla06])
Large Powers, Singularity Analysis and Hayman's Method

Cauchy Integrals - Calculation and Theory

- Dominant Pole Analysis (Chapter 6, [Pem06]; Chapter 6, [Fla06])
- Residue Theorems (Chapter 6, [Pem06]; Chapter 4, [Fla06])
- Coefficient Asymptotics for Meromorphic ODEs (Chapter 7, [Fla06])

Useful Transforms

- Fourier-Laplace Integrals (Chapter 9, [Pem06])
Connections with saddle point and circle methods (steepest descent and Watson's lemma)
- Mellin Transform (Chapter 8, [Szp01]; [Dum95])
- Poissonization and Depoissonization (Chapter 10, [Szp01])

Symbolic Computation

The algebraic/symbolic tools of asymptotic analysis found in sections 4.2 and 4.4 of [Pem05]

References

- [Dum95] Dumas, P., Flajolet, P. & Gourdon, X., "Mellin Transforms and Asymptotics : Harmonic Sums," *Theoretical Computer Science*, vol. 144(1-2) (1995): pp. 3-58.
- [Fla06] Flajolet, P. & Sedgewick, R., *Analytic Combinatorics*, (<http://algo.inria.fr/flajolet/Publications/books.html>, April 13, 2006)
- [Lot04] Lothaire, M. *Applied Combinatorics on Words* (Cambridge University Press, Cambridge, 2004)
- [Pem05] Pemantle, R. & Wilson, M., "Twenty combinatorial examples of asymptotics derived from multivariate generating functions," math.CO/0512548, 2005
- [Pem06] Pemantle, R. *Lecture Notes for Math 581, Spring 2006*
- [Szp01] Szpankowski, W. *Average Case Analysis of Algorithms on Sequences*, (Wiley-Interscience: 2001)