MAT 114 Fall2002 Final Answer Key

Note: Problems marked N/A are not on this year's syllabus (Spring 2007)

Multiple Choice

1. B; 2. D; 3. E; 4. B ; 5. A; 6. D; 7. D; 8. A; 9. D; 10. C; 11. A; 12. E ;13. D; 14. B; 15 1A,2C,3B,4D;

16.a) TRUE: $|(a+b)\times(a-b)| = |a\times a - a\times b + b\times a - b\times b| = 2|b\times a| = 6$

b) TRUE: $i\times a = 0$ implies that $a = c\cdot i$ for some c a real number. $a\cdot i = -1$ implies that $c = -1$, thus $a = -1$ so $|a| = 1$.

17. If $x = y$ then $\frac{sin(x)-sin(y)}{x+y} = 0$ so the limit when $x = y$ and $(x,y)\to(0,0)$ is 0. If $y = 0$ then $\frac{sin(x)-sin(y)}{x+y} = \frac{sin(x)}{x}$ so the limit when $0 = y$ and $(x,y)\to(0,0)$ is 1.

18. The rate of most increase is the direction of the gradient. Thus the gradient of f is $a < 1,1,1 >$. The rate of increase equals to the length of the gradient vector, thus $a\sqrt{3} = 2$ so $a = \frac{2}{\sqrt{3}}$. Now the directional derivative of f in the $u =< 1,-1,1 >$ direction is $\nabla f\cdot\frac{u}{|u|} = \frac{2}{\sqrt{3}} < 1,1,1 > \cdot\frac{1}{\sqrt{3}} < 1,-1,1 >= \frac{2}{3}$.

19. N/A

20. The plane being perpendicular to that line, means that the direction of that line gives us the normal direction to the plane. Now the equation of the plane passing through $(1,-1,1)$ with normal direction $< 0,2,1 >$ is $2(y+1)+z-1 = 0$, which is the same as $2y+z = -1$. We now use the distance formula from a point to a plane to get that the distance from $(1,2,0)$ to our plane is : $\frac{|0\cdot1+2\cdot0+1\cdot2+1|}{\sqrt{0^2+2^2+1^2}} = \frac{3}{\sqrt{5}}$