Information about the First Hour exam for Math 114-001

Mechanics of the exam:

(1) The exam will begin on Monday, Feb. 4, in our usual room, DRL A8, at the usual time (11:00 am). It will be 45 minutes long to give us time to collect the exams before everyone must leave for their next class.

(2) **Important:** You can (and should) bring one handwritten one-sided page of notes to the exam, but no books or calculators. On this page, write down any formulas or notes which you think may help. See the next section for advice on how to prepare for the exam. Remember - no typed or photoreduced notes; only handwritten notes are allowed.

(3) There will be 10 multiple choice questions on the exam. To receive credit for a problem you will have to mark the correct answer **and** show plausible work leading to this answer. There will be no partial credit.

(4) There will be multiple versions of the exam to discourage copying answers from other students.

(5) The exam will be proctored by Professor Alex Popa and our two T.A.'s, Omar and Jamol. Our class on Wednesday and Friday of next week will be taught by Asher Auel.

Scope of the exam and advice on how to prepare for it:

(1) The exam will cover the material in sections 13.1, 13.2, 13.3, 13.4, 13.5 in addition to the beginning of section 14.4, which concerns the position, velocity, acceleration and force vectors associated to a moving particle.

(2) I would recommend trying to become comfortable with how to do all the core homework problems on the syllabus for sections 13.1 through 13.5, in addition to problems 7, 8 and 20 of section 14.4. I'll give the answers to problems 8 and 20 of section 14.4 below, since even numbered problems do not have answers at the back of the book.

(3) Do at least one problem about the fact that the work done by a constant force vector F on a particle moving through a displacement represented by a vector D is the dot product $F \cdot D$. I recommend doing problem 45 of section 13.3.

(4) Do at least one problem dealing with spheres, such as problem 15 on page 833.

(5) Do at least one problem about torque, such as problem 35 on page 857.

(6) Review the yoga for defining a plane by specifying a point $B = (b_1, b_2, b_3)$ on the plane and a normal vector $n = (n_1, n_2, n_3)$ to the plane. If $Q = (x, y, z)$ is a point on the plane, understand why

$$0 = n \cdot (Q - B) = n_1(x - b_1) + n_2(y - b_2) + n_3(z - b_3) = n_1x + n_2y + n_3z - d$$

where

$$d = n \cdot B = n_1 b_1 + n_2 b_2 + n_3 b_3.$$

Notice that this formula tells you that if you are given a linear equation

$$ax + by + cz = d$$

for a plane, then a normal vector to the plane is just $n = (a, b, c)$. See problem 27 on page 866, for example.

(7) If two planes P_1 and P_2 have normal vectors n_1 and n_2, respectively, then the angle θ between the planes is the angle between n_1 and n_2. The formula for the dot product

$$n_1 \cdot n_2 = |n_1||n_2|\cos(\theta)$$

gives a way to compute this angle. See problem 47 on page 867, for example.

(8) Remember that the cross product $n_1 \times n_2$ gives a vector perpendicular to both n_1 and n_2 if these vectors don't lie in the same plane in $\mathbb{R}^3$. Review the right hand rule, the magnitude of $n_1 \times n_2$ and the determinant procedure for finding $n_1 \times n_2$. See problems 3 and 23 on page 856.

(9) Review the different ways to describe equations for lines. See problems 12, 35 and 37 on page 866. (I'll give the answer to problem 12 in the next section.)

(10) Review the method described in class for finding the point Q on a plane P which corresponds to a point D in space as seen by an observer at position A. You should be able to find this Q given A and D, a point B on the plane P and a normal vector n to P. (You won't need to know the derivation of vanishing points.)

(11) A final note on technique. If you are trying to find a plane which satisfies various properties, consider linear equations

$$ax + by + cz = d$$

which define this plane. The properties you are interested in put constraints on a, b, c and d. By writing these down you may be able to find all choices of a, b, c and d which work. Try this for problem 33 on page 866.

Some answers:

(1) Problem 8 of section 14.4. $r(t) = (t, t^2, t^3)$, $v(t) = r'(t) = (1, 2t, 3t^2)$, $a(t) = v'(t) = r''(t) = (0, 2, 6t)$. The speed is the magnitude of acceleration, which is

$$s(t) = |v(t)| = \sqrt{1 + 4t^2 + 9t^4}.$$

So $r(1) = (1, 1, 1)$, $v(1) = (1, 2, 3)$, $a(1) = (0, 2, 6)$, $s(1) = \sqrt{1 + 4 + 9} = \sqrt{13}$.

(2) Problem 20 of section 14.4. The position of the particle is $r(t) = (t^3, t^2, t^3)$ as a function of time t. So its acceleration at time t is $a(t) = r''(t) = (6t, 2, 6t)$. Newton's law says the force acting is $F(t) = ma(t) = m(6t, 2, 6t)$ when m is the mass of the particle.

(3) Problem 12 on page 866. We are to find the line of intersection of the planes $x + y + z = 1$ and $x + z = 0$. A quick way to do this is to treat one of the coordinates as a variable, say the x coordinate, and to then solve for the other coordinates as a function of x. The second plane gives $z = -x$, and the first plane then gives $y = 1 - x - z = 1 - x - (-x) = 1$. So the line has equations $z = -x$ and $y = 1$, and each point on the line can be written as $(x, 1, -x)$ for some x. Notice that all such points solve the equations defining the two planes.

A more geometric approach is to use the normal vectors to the two planes. The plane $x + y + z = 1$ has normal vector $n_1 = (1, 1, 1)$ since the components of this vector are the coefficients of the variables in the equation. The plane $x + z = 0$ has normal vector $n_2 = (1, 0, 1)$. Any line in one of the planes must have direction vector v perpendicular to the normal to the plane. So if v is a direction vector for the intersection of the planes, v is perpendicular to n_1 and to n_2. A way to construct such a v is to take the cross product

$$v = n_1 \times n_2 = (1, 0, -1).$$

(Check this cross product!). If B is any point on the line, then every point on the line has the form $B + tv$ for some real number t. To find a B on the line, we can look for one with x-coordinate 0. Then $B = (0, 1, 0)$ from solving the equations defining the planes for z and y, so the line is the set of points

$$\{(0, 1, 0) + t(1, 0, -1) : t \in \mathbb{R}\}.$$

This agrees with the earlier (simpler) approach.