Math104-007, Fall 2007

Fall06Makeup: the answer key and some comments

1. C) $\int_0^{\pi/8}(\cos(2x) - \sin(2x))dx = \frac{\sqrt{2}-1}{2}$

2. B) $\int_0^2 \pi x^5 dx = \frac{32\pi}{3}$

3. D) $\int_0^{\sqrt{\pi}} 2\pi x \sin(x^2) = 2\pi$

4. F) Twice by parts.

5. A) Trigonometric substitution $x = 2\sqrt{2}\sin t$

6. E) Table integration. Then use that $\lim_{t\to\infty} \tan^{-1}(x) = \frac{\pi}{2}$.

7. B) $V = 2\pi \int_4^5 \frac{xdx}{x^2-5x+6}$. The partial fraction decomposition is $\frac{x}{x^2-5x+6} = \frac{3}{x-3} - \frac{2}{x-2}$, and the rest is easy.

8. E)

$$dS = 2\pi y ds = 2\pi\sqrt{1-x^2}\sqrt{1+\left(\frac{-x}{\sqrt{1-x^2}}\right)^2}dx = 2\pi\sqrt{1-x^2}\sqrt{\frac{1}{1-x^2}}dx = 2\pi dx$$

hence $S = \int_0^{1/2} 2\pi dx = \pi$

9. C) $\int x^{-1.2}dx = -5x^{-0.2}$, hence $\int_{3^5}^{\infty} x^{-1.2}dx = 5(3^5)^{-0.2} - \lim_{x\to\infty} 5x^{-0.2} = 5(3^5)^{-1/5} = 5/3$.

10. A) The point corresponds to $t = 1$. The slope is $\frac{dy/dt}{dx/dt} = \left.\frac{3t^2}{1-2/t^3}\right|_{t=1} = -3$, hence the line is $y = -3x + C$. The point $(2, 4)$ is on the curve, so $4 = -6 + C$.

11. F) The loop is from $t = -2$ to $t = 2$. So, $L = \int_{-2}^{2} \sqrt{(3t^2-4)^2 + (4\sqrt{3}t)^2}dt = \int_{-2}^{2}(3t^2+4)dt = 32$

12. A) A leaf is from $\pi/6$ to $\pi/2$ (or from $-\pi/2$ to $\pi/2$), hence

$$A = \int_{\pi/6}^{\pi/2} \frac{1}{2} r^2 d\theta = \int_{\pi/6}^{\pi/2} \frac{9}{2} \cos^2(3\theta) d\theta$$

Then use that $\cos^2 x = \frac{1+\cos 2x}{2}$.

13. E) Apply L'H rule twice.

14. D) Notice that

$$\ln(2n+1) - \frac{1}{2}\ln(n^2+1) = \ln\left(\frac{2n+1}{\sqrt{n^2+1}}\right)$$

and

$$\lim_{n\to\infty} \frac{2n+1}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{2n}{\sqrt{n^2}} = 2$$

15. C) The series is a sum of two geometric progressions: $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \ldots$ and $\frac{3}{5} + \frac{9}{25} + \frac{27}{125} + \ldots$.

16. C) I is a convergent geometric series; II is a divergent p-series; III is a divergent geometric series; IV is a sum of two convergent geometric series, so it converges; the limit of a_n's in V is 1, hence the series diverges.

17. B) I diverges because $\lim_{n\to\infty} \frac{n^2}{1+n^2} = 1$; II absolutely converges because we can compare it by the limit comparison to $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (p-series with $p = 2$); III conditionally converges by the alternating test, the series of the absolute values $\sum_{n=1}^{\infty} \frac{n \ln n}{1+n^2}$ diverges because $\frac{n \ln n}{1+n^2} > \frac{1}{n}$ and the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

18. D) By ratio test $|2x-5| < 4$, hence we have absolute convergence on $(\frac{1}{2}; \frac{9}{2})$. Checking the end-points, we see that the series also converges at both end-points.

19. F) Straightforward: substitute x^2 into the Maclaurin series of $\tan^{-1} x$.

20. B) Straightforward: substitute $\sqrt{x}$ into the Maclaurin series of $\cos x$ and integrate term by term.