Mathematics 104-007, Fall 2007

Sample midterm

The recommended duration of the exam is 45 minutes. There are eleven multiple choice questions. Questions are worth 10 (eight possible answers) or 5 (three possible answers) points. The total is 90 points, but the resulting grade cannot exceed 80. As usual, problem 11 is a difficult bonus problem, and you can get the full score without solving it.

1. a) b) c) d) e) f) g) h)
2. a) b) c) d) e) f) g) h)
3. a) b) c) d) e) f) g) h)
4. a) b) c)
5. a) b) c)
6. a) b) c)
7. a) b) c)
8. a) b) c) d) e) f) g) h)
9. a) b) c) d) e) f) g) h)
10. a) b) c) d) e) f) g) h)
11. a) b) c) d) e) f) g) h)

1. [**10 points**] Find the limit of the sequence

$$\lim_{n\to\infty} n^{\frac{1}{n}}$$

a) $\sqrt{e}$; b) π; c) e^2; d) 1;
e) 2; f) e; g) $\frac{\pi}{2}$; h) the limit does not exist.

2. [**10 points**] Find the limit of the sequence

$$\lim_{n\to\infty} n \sin \frac{2}{n}$$

a) e; b) 0; c) 2; d) $\frac{\pi}{2}$;
e) 1; f) $e - \frac{\pi}{2}$; g) e^2; h) the limit does not exist.

3. [**10 points**] Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{1-(-1)^n}{5^{n-1}}$$

a) $\frac{25}{12}$; b) $\frac{10}{3}$; c) $\frac{9}{4}$; d) 2;
e) $\frac{8}{3}$; f) $\frac{25}{24}$; g) $\frac{5}{2}$; h) the series diverges.

4. [**5 points**] The series

$$\sum_{n=1}^{\infty} \frac{n \ln n - \sqrt{n}}{n^2 \ln n + n\sqrt{n}}$$

a) absolutely converges;
b) conditionally converges;
c) diverges.

5. [**5 points**] The series

$$\sum_{n=1}^{\infty} \frac{2^n n!}{(2n)!}$$

a) absolutely converges;
b) conditionally converges;
c) diverges.

6. [**5 points**] The series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$$

a) absolutely converges;
b) conditionally converges;
c) diverges.

7. [**5 points**] The series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt[10]{n}}{(\ln n)^{10}}$$

a) absolutely converges;
b) conditionally converges;
c) diverges.

8. [**10 points**] Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n3^n}$$

a) $[-2,1]$; b) $(-1,2]$; c) $[-2,1)$; d) $[-2,2]$;
e) $(-1,\frac{1}{2})$; f) $(-\infty,\infty)$; g) $(-2,1)$; h) $(-1,1)$.

9. [**10 points**] Use an appropriate series to calculate the indefinite integral

$$\int_0^{0.1} \frac{dx}{1+x^4}$$

to six decimal digits.

a) 0.099995; b) 0.099998; c) 0.099999; d) 0.100000; e) 0.100001; f) 0.100002; g) 0.100005; h) the integral diverges.

10. [**10 points**] Find the first three terms of the Maclaurin series of

$$\int_0^x \frac{\sin t dt}{t}$$

a) $\frac{x^2}{2} - \frac{x^3}{3\cdot 2!} + \frac{x^4}{4\cdot 3!}$; b) $\frac{x}{2} - \frac{x^2}{2\cdot 3!} + \frac{x^5}{2\cdot 5!}$;
c) $1 - \frac{x}{2!} + \frac{x^2}{4!}$; d) $x - \frac{x^3}{(2!)^2} + \frac{x^5}{(4!)^2}$;
e) $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$; f) $1 - \frac{x^2}{2} + \frac{x^4}{4}$;
g) $1 - \frac{x^2}{2\cdot 2!} + \frac{x^4}{4\cdot 4!}$; h) $x - \frac{x^3}{3\cdot 3!} + \frac{x^5}{5\cdot 5!}$.

11. [**10 points**] Find the sum of the series

$$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \dots$$

a) $\frac{1}{\ln 2}$; b) e^{-2}; c) $\ln \ln 2$; d) $\frac{1}{2}$;
e) $\frac{1}{\sqrt{2}}$; f) $\frac{1}{\sqrt{e}}$; g) $\frac{1}{(\ln 2)^2}$; h) the series diverges.