Math114, Fall 2007

Spring05: an answer key with some comments

True/False.

1. T. (Though, strictly speaking, one has to require that $f(x, y)$ is differentiable!)
2. T. Local max ($f(x, y)$ is of the form $g(x)h(y)$, where g has a local max at x_0 and h has a local max at y_0, and both are positive).
3. F. Not a critical point.
4. T. Local min ($f(x, y)$ is of the form $g(x)+h(y)$, where g has a local min at x_0 and h has a local min at y_0). Notice that $f_{xx}f_{yy} - f_{xy}^2 = 0$ at (x_0, y_0), but we can solve the problem because of a special form of $f(x, y)$.
5. T. Saddle ($f(x, y)$ is of the form $h(y) - g(x)$, where g has a local min at x_0 and h has a local min at y_0). Notice that $f_{xx}f_{yy} - f_{xy}^2 = 0$.
6. F.
7. T. (One has $x^2 + y^2 = 1$. This is a standard example of an alternative parameterization of the unit circle.)
8. F. (Any initial value problem has unique solution, in particular, the graphs of different solutions of an ode do not intersect.)
9. T.
10. T. (Though we have not studied so-called exact ode's, and they are not in our syllabus.)

Multiple choice.

The first four are easy: 1. b); 2. c); 3. a); 4. d)

5. f) $r_1 = r_2 = -5$, hence xe^{-5x}

6. d) Polar substitution is very natural here. The region of integration is a bottom half of a unit circle, hence the original integral equals to $\int_\pi^{2\pi} \int_0^1 \frac{4r}{1+r^2} r dr d\theta = 4\pi - \pi^2$.

7. b) Easy: $r_{1,2} = 1 \pm \sqrt{3}$.

8. a) A very typical equation. Here $r_{1,2} = \pm\lambda i$

9. e) The tangent vector at time t is $(2t-1, 2t+1)$, and it is horizontal when $2t+1=0$.

10. e) $\int_0^{2\pi}\int_0^1 r^2 dr d\theta = 2\pi/3$.

11. Not in our syllabus (exact differential equations).

12. c) $\mathbf{r}'(\pi/2) = (-e^{\pi}, 2e^{\pi}, 2e^{\pi})$, which is parallel to $(-1,2,2)$. Normalizing the latter vector, we get $(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$.

Free response.

2. $r_{1,2} = \frac{1\pm\sqrt{7}i}{2}$, hence $y = Ae^{x/2}\cos(\sqrt{7}x/2) + Be^{x/2}\sin(\sqrt{7}x/2)$ is the general solution. To find A, B we have to express $y(0)$ and $y'(0)$ using A and B, and solve the obtained linear system on A and B. The obtained answer is $y = e^{x/2}\cos(\sqrt{7}x/2) + \frac{7}{\sqrt{7}}e^{x/2}\sin(\sqrt{7}x/2)$.

3. One has to maximize the volume $\frac{2\pi r^3}{3} + \pi r^2 h$ given the budget constraint $40\pi r^2 + 20\pi rh = 8000$. One can do it straightforwardly by substituting $h = (400 - 2\pi r^2)/\pi r$ into the formula of the volume and then maximizing on r. Another possibility is to use Lagrange multipliers. All in all, one obtains that $r = \frac{10}{\sqrt{\pi}}$ and $h = \frac{20}{\sqrt{\pi}}$.

4. A standard guess is $y_p = Ax^2 + Bx + C$. Solving a system of linear equations one gets $y_p = x^2/16 + x/16 + 3/128$. Since $r_1 = r_2 = 4$, $y = y_p + ae^{4x} + bxe^{4x}$, and solving for a and b (use the initial condition) one gets $a = 125/128$ and $b = -127/32$.

My personal opinion is that a better choice of constants would serve both the students during the exam and the grading team after the exam.

5. So-called homogeneous equations are not in the current syllabus (for the sake of completeness, we note that the trick here is to substitute $z(x) = y(x)/x$, then $y' = (xz)' = xz' + z$ and you can check as an exercise that this substitution produces a separable first-order ode on $z(x)$).