Instructions: *There are twenty-three questions on this exam, of which you need only to do twenty in order to obtain full marks.*

1. Consider four functions a, b, c, d whose assignment rules are given by

$$a(x,y) = x - y, \quad b(x,y) = x + y, \quad c(x,y) = x^2 - y, \quad d(x,y) = x - y^2,$$

and the four contour plots I, II, III, and IV in figures 1 through 4. Which of the following matches is correct?

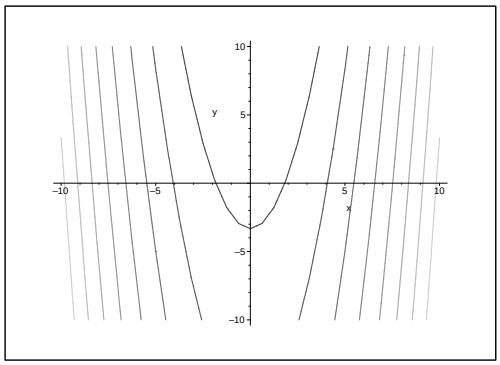

Figure 1: I

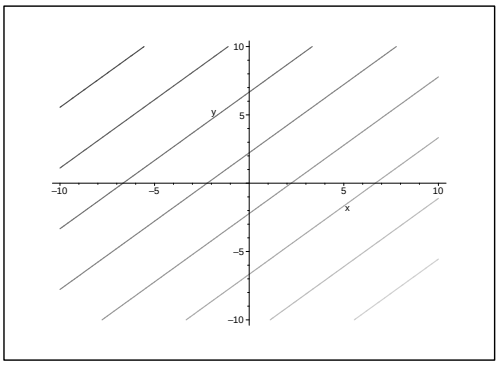

Figure 2: II

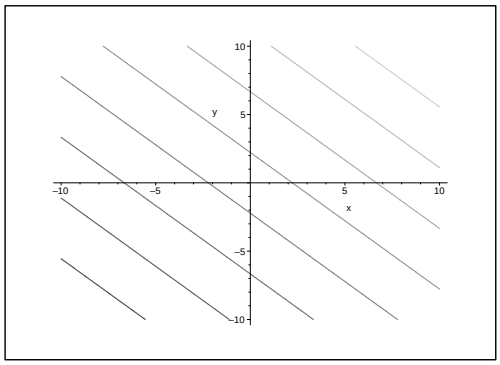

Figure 3: III

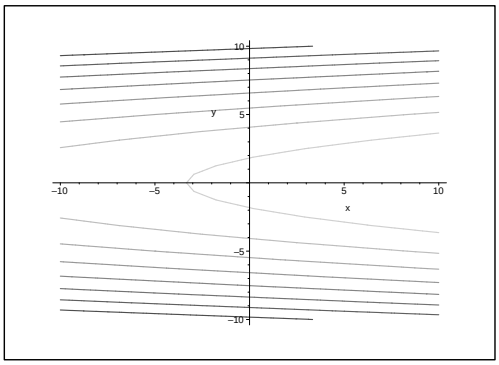

Figure 4: IV

(A) (a, III), (b, II), (c, I), (d, IV)

(B) (a, II), (b, III), (c, I), (d, IV)

(C) (a, III), (b, II), (c, IV), (d, I)

(D) (a, II), (b, III), (c, IV), (d, I)

(E) none of these

2. Let $f(x,y) = \dfrac{x+y}{x^2-y}$. Which of the following assertions is false?

   (A) $\lim\limits_{\substack{(x,y)\to(0,0)\\ y=0}} f(x,y) = 1$

   (B) $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist

   (C) $\lim\limits_{\substack{(x,y)\to(0,0)\\ y=x}} f(x,y) = -2$

   (D) $\lim\limits_{\substack{(x,y)\to(0,0)\\ x=0}} f(x,y) = -1$

   (E) all of them are true

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3. Which of the following limits is correct?

   (A) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{(y-1)(x^2+y^2)} = 0$

   (B) $\lim\limits_{(x,y)\to(1,2)} \dfrac{-4x+2}{1-x+y^2} = 2$

   (C) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{x^2y} = 0$

   (D) $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x-y^2}{2x^2+y} = 1$

   (E) all are incorrect

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4. Find the tangent plane to the surface $x^2+y^2-z^2=0$ at the point $(3,4,5)$.

   (A) $3x+4y-5z=0$ (B) $3x-y-5=0$ (C) $2x+y-z-5=0$ (D) $3x-2y-1=0$ (E) $x+2y-2z-2=0$

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5. Find the point at which the tangent plane to the surface $2x^2+xy+y^2+4x+8y-z+14=0$ is $4x+y-z=0$.

   (A) $(1,-4,0)$ (B) $(1,-3,2)$ (C) $(0,-4,-2)$ (D) $(0,-3,-1)$ (E) $(1,-4,-1)$

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6. Let $h(x,y) = x\sqrt{y}$ and observe that $h(1,4)=2$. Approximate $h(0.9,4.1)$ through differentials.

   (A) $2-\dfrac{7}{40}$ (B) $2+\dfrac{7}{40}$ (C) $2+\dfrac{9}{40}$ (D) $2-\dfrac{1}{40}$ (E) none of these

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7. Let $z=f(x,y)$ be implicitly defined by the equation $x^y+xz^2=2$. Find $\dfrac{\partial z}{\partial x}$ in a neighbourhood of $(1,1,1)$.

   (A) $-\dfrac{5}{4}$ (B) -1 (C) 1 (D) $-\dfrac{1}{4}$ (E) none of these

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8. Let $f:\mathbb{R}^3\to\mathbb{R}$ a function all whose first order partial derivatives are continuous and such that $f(0,1,1)=0$, $f_x(0,1,1)=1$, $f_y(0,1,1)=2$, $f_z(0,1,1)=3$. Find $\lim\limits_{t\to 0}\dfrac{f(t^2,\cosh t,e^t)}{f(t,\cos t,\cosh t)}$.

   (A) 3 (B) 2 (C) $\dfrac{1}{3}$ (D) -3 (E) none of these

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9. Let $f:\mathbb{R}^2\to\mathbb{R}$ such that $f(x,y)=f(y,-x)$ for all $(x,y)\in\mathbb{R}^2$, and $f(2,y)=2+|y|$. Then

   (A) $f_x(1,2)=1$ (B) $f_x(1,2)=0$ (C) $f_x(0,2)=1$ (D) $f_x(0,2)=-1$ (E) none of these

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10. Let $z = 2x + y^2$ and $x = 3t, \quad y = 2t$. Determine $\dfrac{\mathrm{d}z}{\mathrm{d}t}$.

(A) $6 + 6t$ (B) $6 + 8t$ (C) $2 + 2t$ (D) $4 + 12t$ (E) none of these

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11. If $(1, 2)$ is a local minimum of a continuously differentiable function f, subject to the constraint $x - y = -1$, which of the following assertions is true?

(A) there is a real number λ such that $\nabla f(1, 2) = (\lambda, -\lambda)$

(B) for all $(x, y) \neq (1, 2)$ in a neighbourhood of $(1, 2)$ we have $f(x, y) > f(1, 2)$

(C) $f(x, y) \geq f(1, 2)$ for all $y = x + 1$

(D) $(1, 2)$ is a critical point of f

(E) none of these is true

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12. Which of the following is a critical point for $f(x, y) = 6x^2 + 6y^2 + 6xy + 36x - 54y - 5$?

(A) $(-7, 8)$ (B) $(8, -7)$ (C) $(7, -8)$ (D) $(-8, 7)$ (E) none of these

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13. Let $f(x, y, z) = \dfrac{ax^2}{2} - x + \dfrac{y^3}{3} + yz^2 - b^2y$. Which of the following statements is false?

I $(\dfrac{1}{a}, b, 0)$ is a minimum if and only if $ab > 0$.

II If $a = 0$, f does not have saddle points.

III If $ab > 0$, $(\dfrac{1}{a}, 0, -b)$ is a saddle point.

IV If $b = 0$ and $a \neq 0$, $(\dfrac{1}{a}, 0, 0)$ is the only critical point of f.

(A) I only (B) II only (C) III only (D) IV only (E) all are true

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14. The product of the maximum and minimum values of the function $f(x, y) = xy$ on the ellipse $\dfrac{x^2}{9} + y^2 = 2$ is

(A) -9 (B) 9 (C) 12 (D) -12 (E) none of these

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15. Find the maximum of the function $f(x, y) = 2x + y$ subject to the constraint $2x^2 + y^2 = 3$.

(A) 3 (B) $\sqrt{3}$ (C) $2\sqrt{3}$ (D) 1 (E) none of these

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16. $f(x, y) = -x^3 - y^3 + 9xy - 6$ has a local maximum and a saddle point. Find the value of f at the saddle point.

(A) -6 (B) -87 (C) 21 (D) 14 (E) none of these

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17. Let f be a function for which $\int_2^4 \int_x^{2x} f(x,y)\mathrm{d}y\mathrm{d}x$ exists. Which of the following expressions is equal to this integral with the order of integration reversed? You must draw the region of integration in order to receive credit.

(A) $\int_2^4 \int_2^y f(x,y)\mathrm{d}x\mathrm{d}y + \int_4^8 \int_{y/2}^4 f(x,y)\mathrm{d}x\mathrm{d}y$

(B) $\int_x^{2x} \int_2^4 f(x,y)\mathrm{d}x\mathrm{d}y$

(C) $\int_2^3 \int_{y/2}^y f(x,y)\mathrm{d}y\mathrm{d}x$

(D) $\int_2^4 \int_{y/2}^4 f(x,y)\mathrm{d}x\mathrm{d}y + \int_4^8 \int_4^y f(x,y)\mathrm{d}x\mathrm{d}y$

(E) none of these

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18. Find $\int_0^2 \int_0^1 x\sin(xy)\mathrm{d}y\mathrm{d}x$

(A) $2-\sin 2$ (B) $2+\sin 2$ (C) 0 (D) 2 (E) none of these

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19. Find $\iint_S x\mathrm{d}A$ where S is the region bounded by $y=x^2$ and $x=y^2$ in the first quadrant.

(A) $\frac{3}{20}$ (B) $\frac{3}{10}$ (C) $\frac{11}{5}$ (D) $-\frac{3}{20}$ (E) $\frac{1}{20}$

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20. Let S be the interior and boundary of the triangle with vertices $(0,0)$, $(1,1)$, and $(2,0)$. Find $\iint_S y\mathrm{d}A$.

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) 1 (E) none of these

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21. Find $\int_0^2 \int_0^1 x^3 y e^{x^2y^2}\mathrm{d}x\mathrm{d}y$.

(A) $\frac{e^4-5}{16}$ (B) $\frac{9e^4-1}{72}$ (C) $\frac{e^4-4}{4}$ (D) $\frac{9e^4-1}{18}$ (E) none of these

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22. Let $\mathscr{R}$ be the region in the first quadrant bounded by the unit circle $x^2+y^2=1$. Find $\iint_{\mathscr{R}} (x^2y+xy^2)\mathrm{d}A$.

(A) $\frac{2}{15}$ (B) $\frac{1}{5}$ (C) $\frac{1}{3}$ (D) $\frac{1}{18}$ (E) none of these

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23. Let $\mathscr{R}$ be the region in the first quadrant bounded by the lines $y=x$ and $y=2x$, and the unit circle $x^2+y^2=1$. The area of this region is given by which one of the following integrals?

(A) $\int_{\pi/4}^{\arctan 2} \int_0^1 r\mathrm{d}r\mathrm{d}\theta$ (B) $\int_{\pi/4}^{\pi/8} \int_0^1 r\mathrm{d}r\mathrm{d}\theta$ (C) $\int_0^1 \int_x^{2x} \mathrm{d}y\mathrm{d}x$ (D) $\int_{\sqrt{2}/2}^{\sqrt{2}} \int_0^{\sqrt{1-x^2}} \mathrm{d}y\mathrm{d}x$ (E) none of these