Solutions to Homework 2

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8.4

#5. 2.
#7. 10.
#13. $\lambda^2 - 3\lambda - 4$.
#25. -104.
#29. $\lambda = -5, 7$.

8.5

#4. Theo. 8.11 (interchange R_2 and R_3) and Theo. 8.12 (divide new R_3 by 2).
#7. Theo. 8.10 (C_2 is a column of zeroes).
#8. Theo. 8.12 (divide C_2 by 2) and Theo. 8.9 (the new C_2 and C_3 are the same).
#9. Theo. 8.8 ($\det \mathbf{A} = \det \mathbf{A}^T$).
#22. $\det \mathbf{A}^2 = \det \mathbf{AA} = (\det \mathbf{A})(\det \mathbf{A})$ (By Theo. 8.13, page 380).
So, by above we get $\det \mathbf{A}^2 = (\det \mathbf{A})^2$.
But again as $\mathbf{A}^2 = \mathbf{I}$, we get $\det \mathbf{A}^2 = \det \mathbf{I} = 1$ (because the identity matrix $\mathbf{I}$ is a triangular/ diagonal matrix and so its determinant is given by the product of its diagonal elements, which is 1).
So, using the above two, we get $(\det \mathbf{A})^2 = 1$ implying $\det \mathbf{A} = \pm 1$.

#23.

$$\begin{vmatrix} a & a+1 & a+2 \\ b & b+1 & b+2 \\ c & c+1 & c+2 \end{vmatrix} \xrightarrow{C_2\to(C_2-C_1)} \begin{vmatrix} a & 1 & a+2 \\ b & 1 & b+2 \\ c & 1 & c+2 \end{vmatrix} \xrightarrow{C_3\to(C_3-C_1)} \begin{vmatrix} a & 1 & 2 \\ b & 1 & 2 \\ c & 1 & 2 \end{vmatrix}$$

$$\xrightarrow{\text{Dividing } C_3 \text{ by } 2} 2\begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} = 0. \text{ (By Theo. 8.9.)}$$

#31. 16.
#33.

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \xrightarrow{R_2\to(R_2-aR_1)} \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ a^2 & b^2 & c^2 \end{vmatrix} \xrightarrow{R_3\to(R_3-a^2R_1)} \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

$$\xrightarrow{R_3\to(R_3-(b+a)R_2)} \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & (c^2-a^2)-(c-a)(b+a) \end{vmatrix}$$

$$\begin{aligned} &= 1(b-a)((c^2-a^2)-(c-a)(b+a)) \text{ (Why?)} \\ &= (b-a)((c-a)(c+a)-(c-a)(b+a)) \\ &= (b-a)(c-a)((c+a)-(b+a)) \\ &= (b-a)(c-a)(c-b). \end{aligned}$$