Solutions to Homework 3

January 27, 2007

8.6

#5. Nonsingular as determinant equals 12, which is nonzero. The inverse is

$$\begin{pmatrix} \frac{1}{6} & 0 \\ \frac{1}{4} & \frac{1}{2} \end{pmatrix}.$$

#9. Nonsingular as determinant equals -30, which is nonzero. The inverse is

$$\begin{pmatrix} \frac{7}{15} & \frac{-13}{30} & \frac{-8}{15} \\ \frac{1}{15} & \frac{-2}{15} & \frac{1}{15} \\ \frac{2}{15} & \frac{7}{30} & \frac{2}{15} \end{pmatrix}.$$

#17. The matrix is noningular and the inverse is

$$\begin{pmatrix} \frac{-1}{4} & \frac{1}{4} \\ \frac{5}{12} & \frac{-1}{12} \end{pmatrix}.$$

#19. The matrix is singular as we get a row of zeroes on the left.

#27. $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ (by Theo. 8.17(ii)) and so we get

$$(\mathbf{AB})^{-1} = \begin{pmatrix} \frac{-1}{3} & \frac{1}{3} \\ \frac{-17}{12} & \frac{55}{12} \end{pmatrix}.$$

#35. To show $\mathbf{AB}$ is nonsingular (by Theo. 8.19), we have to show that $\det \mathbf{AB} \neq 0$. Now, we know that $\det \mathbf{AB} = (\det \mathbf{A})(\det \mathbf{B}) \neq 0$ (Why?). hence $\mathbf{AB}$ is nonsingular.

#36. We know that $\mathbf{AA}^{-1} = \mathbf{I}$. Therefore we have $\det \mathbf{AA}^{-1} = \det \mathbf{I}$. Now, LHS $= (\det \mathbf{A})(\det \mathbf{A}^{-1})$. And, RHS $= 1$. So, we get $(\det \mathbf{A})(\det \mathbf{A}^{-1}) = 1$, implying $\det \mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}}$.

8.7

#1. $x_1 = \frac{-3}{5}$, $x_2 = \frac{6}{5}$.
#7. $x_1 = -4$, $x_2 = 4$, $x_3 = -5$.

8.8

#4. K_2 is an eigenvector with eigenvalue $2i$.

#11. The eigenvalues are $3i$ and $-3i$ and their corresponding eigenvectors are

$$\begin{pmatrix} (1-3i)/5 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} (1+3i)/5 \\ 1 \end{pmatrix} \text{ respectively.}$$

Note: Please note that any nonzero multiple of the eigenvectors are also eigenvectors, and so there is no unique answer when asked for an eigenvector.

#15. The eigenvalues are 0, 4 and -4 and their corresponding eigenvectors are

$$\begin{pmatrix} 1 \\ 5 \\ 25/9 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 9 \\ 1 \end{pmatrix} \text{ respectively.}$$

#18. The eigenvalues are 1 and 3 and their corresponding eigenvectors are

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \text{ respectively.}$$

#19. The eigenvalues are -1, i and $-i$ and their corresponding eigenvectors are

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} i \\ 1 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -i \\ 1 \\ 1 \end{pmatrix} \text{ respectively.}$$