Math 114-004, Fall 2009

Tong Zhu

Department of Mathematics
University of Pennsylvania

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Cylinders and Quadric Surfaces in 3D

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Quadric Surfaces: equation

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

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$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

To sketch the graph of a surface, it is useful to determine the curves of intersection of the surface with planes parallel to the coordinate planes. These curves are called **traces** of the surface.

- Cylinders: a surface that contains of all lines (called **rulings**) that are parallel to a given line and pass through a given plane curve.

Example 1 $x^2 + y^2 = 1$

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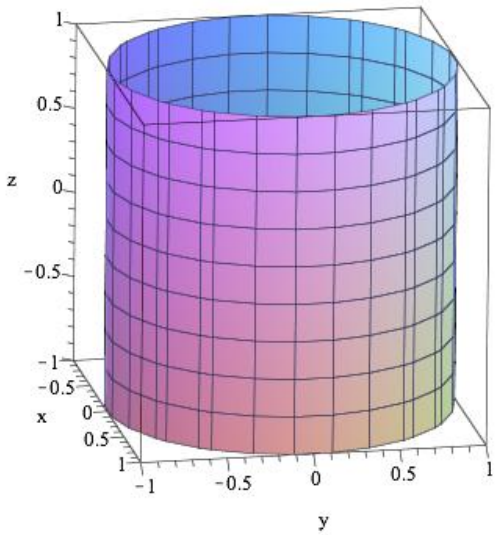
Example 1 $\quad x^2 + y^2 = 1$



Example 2 $x^2 + z^2 = 1$

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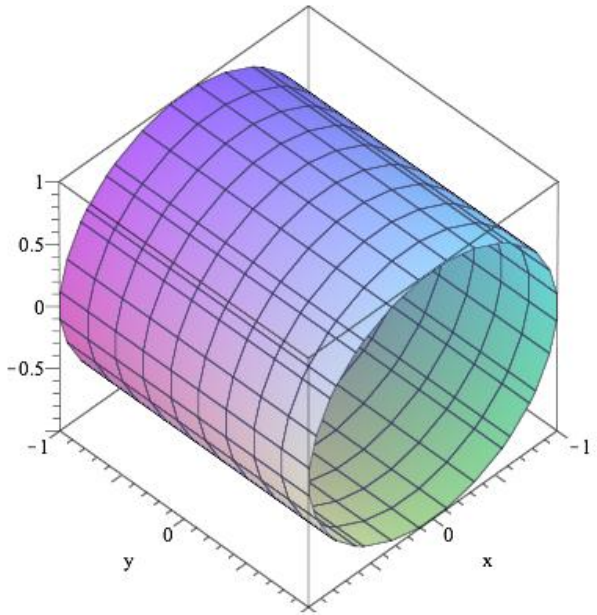

This is also a cylinder.

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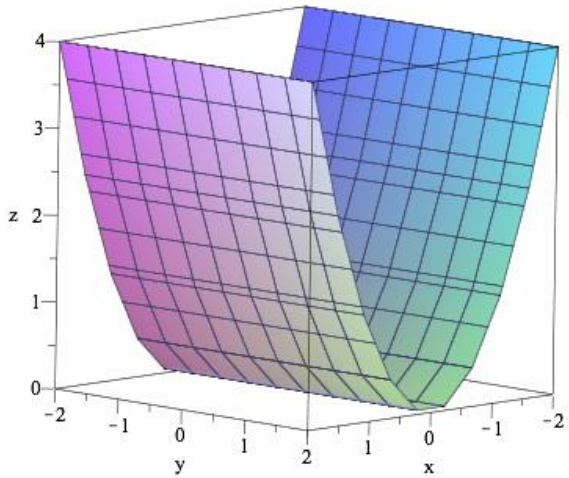

- Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Example 4 $\quad \frac{x^2}{4} + y^2 + \frac{z^2}{16} = 1$

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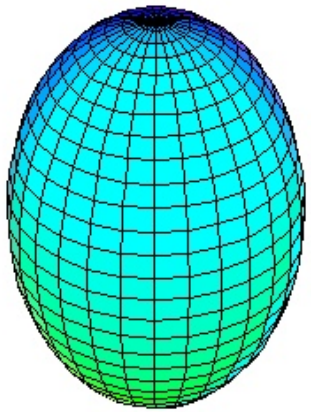

- Elliptic Paraboloid: $\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

Example 5 $z = \frac{x^2}{4} + \frac{y^2}{9}$

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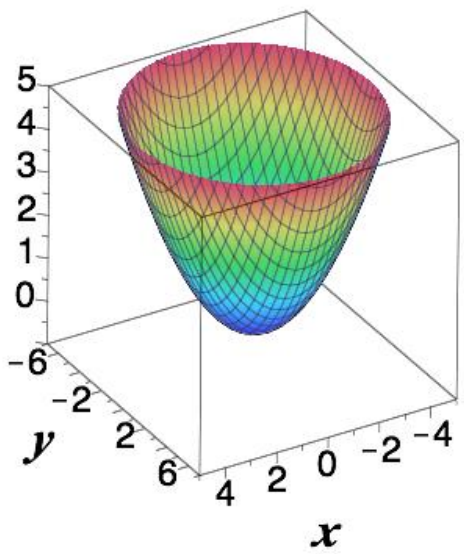

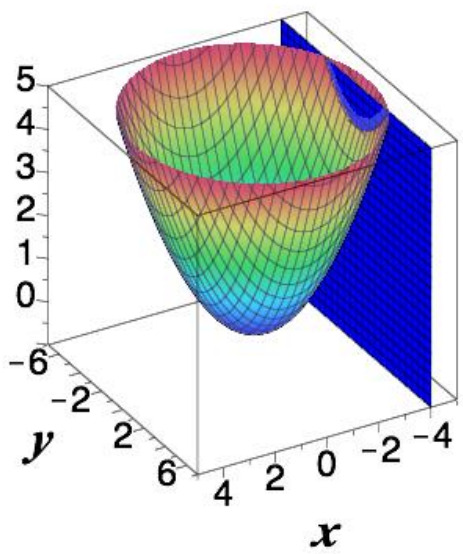

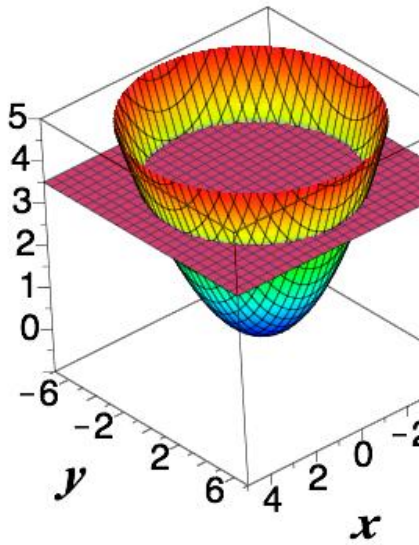

Maple Demonstration

- Hyperbolic Paraboloid: $\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$

Example 5 $$\frac{z}{2} = \frac{x^2}{4} - \frac{y^2}{9}$$

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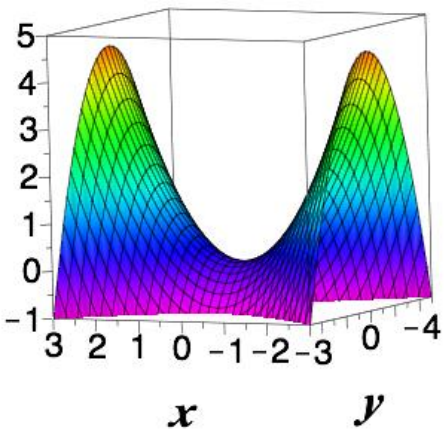

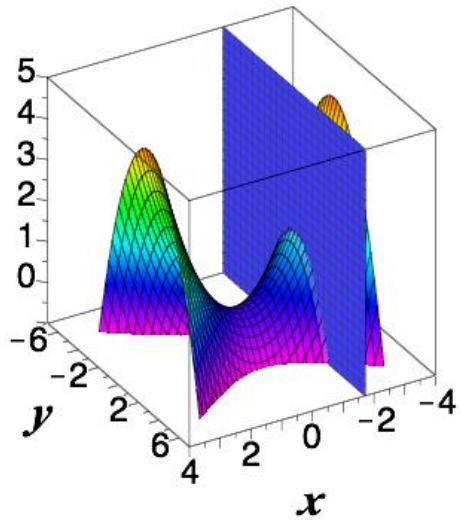

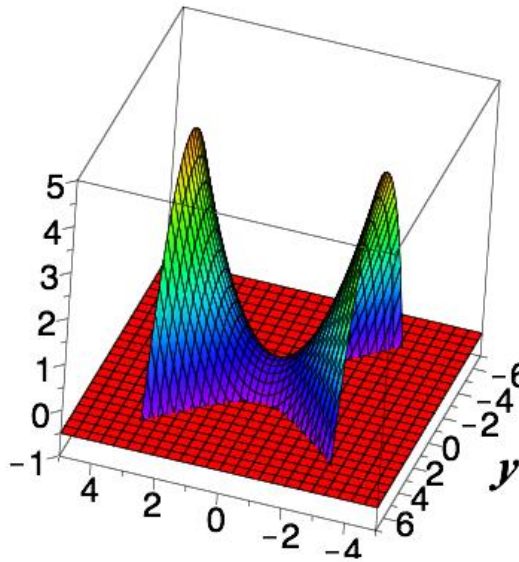

Maple Demonstration

- Cones: $\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

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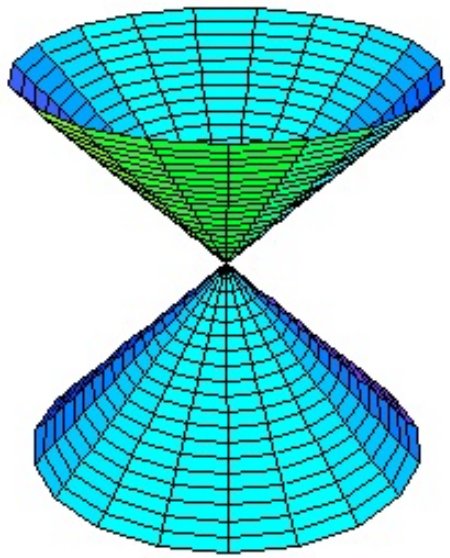

- Hyperboloid of One Sheet: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

Example 6 $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{2} = 1$

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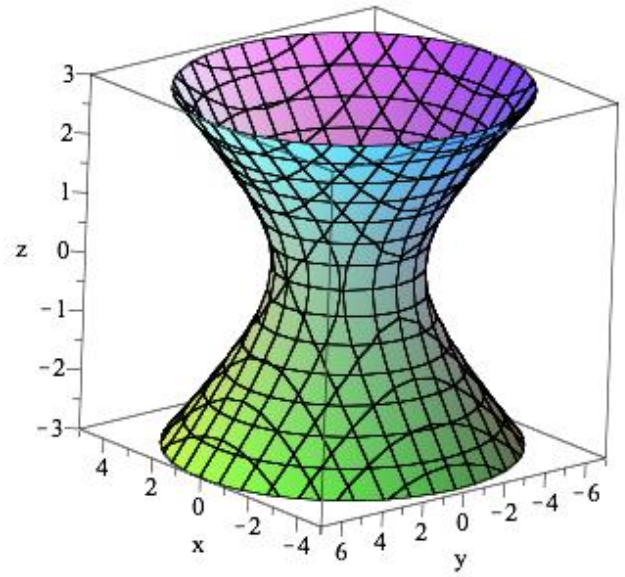

- Hyperboloid of Two Sheets: $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Example 7 $$-\frac{x^2}{4} - \frac{y^2}{9} + \frac{z^2}{2} = 1$$

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