

# Math 114-004, Fall 2009

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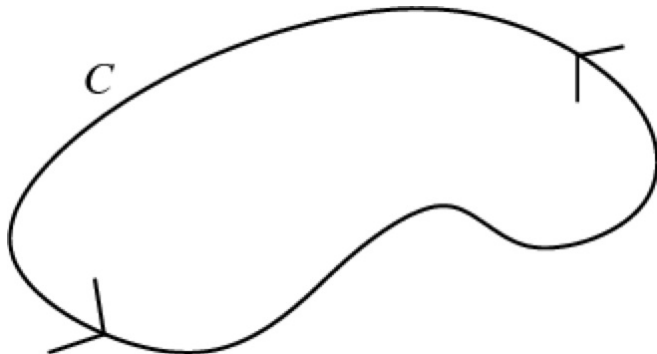
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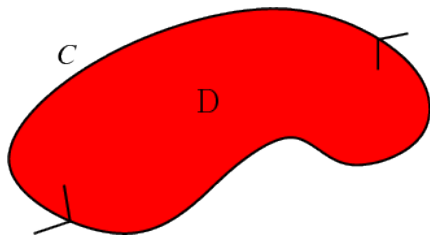
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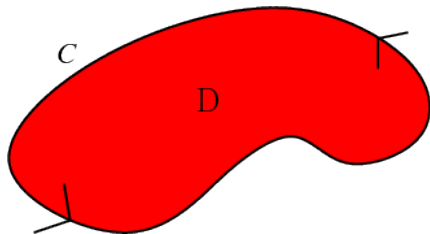
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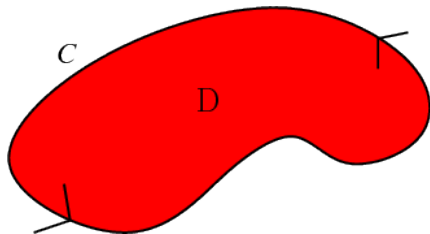






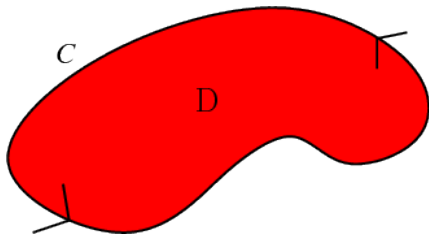
## Theorem (Green's Theorem)

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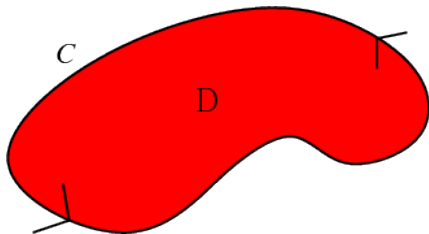
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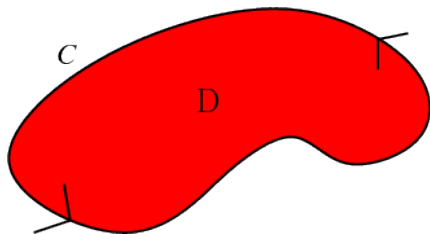
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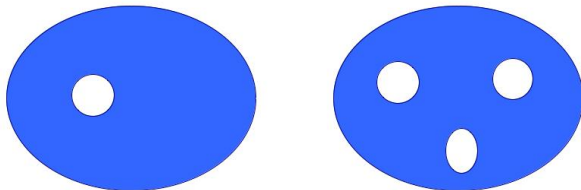
Other notations for closed curve:  $\oint_C Pdx + Qdy$

**Example 1** Evaluate  $\oint_C xydx + x^2y^3dy$  where  $C$  is the triangle with vertices  $(0, 0)$ ,  $(1, 0)$  and  $(1, 2)$  and is positively oriented.

Green's Theorem does not only work for **simple region** like



but also works for regions with holes, like



**Example 2** Evaluate  $\oint_C y^3 dx - x^3 dy$ , where  $C$  is the union of the two circles of radius 1 and radius 2 centered at the origin with positive orientation.

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