A Note on #22 in Section 6.5

The original problem asks the average velocity with respect to r, the distance from the central axis. But in many applications, it is more meaningful to consider the average with respect to the cross-sectional area. How to find that given the same information as in #22?

Let $A = \pi R^2$ and $a = \pi r^2$. Then we can rewrite $v(r)$ as a function of a.

$$v(a) = \frac{P}{4\pi\eta l}(A - a), \qquad a \in [0, A]$$

So the average with respect to the cross-sectional area is

$$\begin{aligned} v_{\text{ave}} &= \frac{1}{A}\int_0^A \frac{P}{4\pi\eta l}(A - a)da \\ &= \frac{P}{4\pi\eta l A}(Aa - \frac{1}{2}a^2)|_0^A \\ &= \frac{PA}{8\pi\eta l} \end{aligned}$$

If it is asked to use the given parameter R, this average is

$$v_{\text{ave}} = \frac{PR^2}{8\eta l}$$