Math 114-004, Fall 2009

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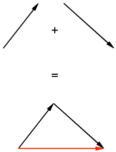
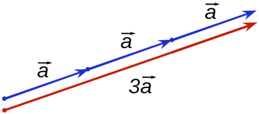
Vector Algebra

- Vector Addition



- Scalar Multiplication



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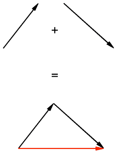
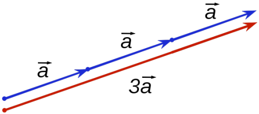
- Vector Addition 

- Scalar Multiplication 

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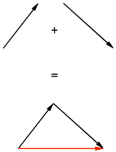

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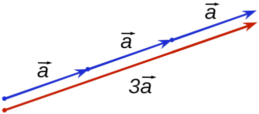

- How about the product of two vectors?
  1. Dot Product $\vec{v} \cdot \vec{u}$
  2. Cross Product $\vec{v} \times \vec{u}$

Before the geometry,

Determinant

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- Determinant of a 2×2 matrix,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

- Determinant of a 3×3 matrix,

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

Example 1

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$$\begin{vmatrix} 2 & 4 & 6 \\ -1 & 3 & 5 \\ 7 & 2 & 6 \end{vmatrix} = 2\begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix} - 4\begin{vmatrix} -1 & 5 \\ 7 & 6 \end{vmatrix} + 6\begin{vmatrix} -1 & 3 \\ 7 & 2 \end{vmatrix}$$

$$=2(18 - 10) - 4(-6 - 35) + 6(-2 - 21) = 42$$

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Cross Product

In 3-D,

Definition If two vectors $\vec{a} =< a_1, a_2, a_3 >$ and $\vec{b} =< b_1, b_2, b_3 >$, then the **cross product** of $\vec{a}$ and $\vec{b}$ is defined as

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But the components of this vector is hard to memorize. There must be a neat formula!

By the notation of determinant, if $\vec{a} = < a_1, a_2, a_3 >$ and $\vec{b} = < b_1, b_2, b_3 >$,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Example $\vec{a} = < 2, 3, 4 >$, $\vec{b} = < 1, 5, -6 >$

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$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 4 \\ 1 & 5 & -6 \end{vmatrix} \\ &= \begin{vmatrix} 3 & 4 \\ 5 & -6 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & 4 \\ 1 & -6 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} \vec{k} \\ &= -38\vec{i} + 16\vec{j} + 7\vec{k} \end{aligned}$$

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- $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

Geometric Aspect of Cross Product

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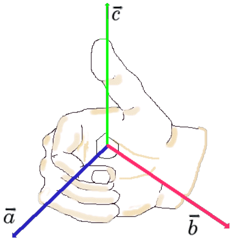
$\vec{a} \times \vec{b}$ is a vector $\perp$ to both $\vec{a}$ and $\vec{b}$.



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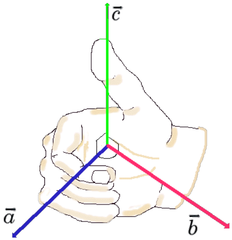

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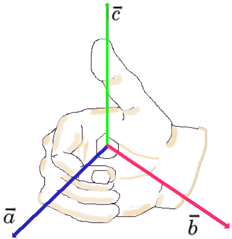
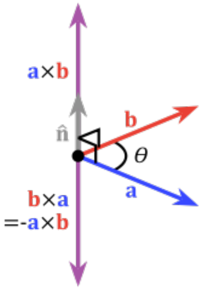


Proof: Compute the dot product

$$(\vec{a} \times \vec{b}) \cdot \vec{a} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} a_1 - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} a_2 + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} a_3$$
$$= a_1(a_2 b_3 - a_3 b_2) - a_2(a_1 b_3 - a_3 b_1) + a_3(a_1 b_2 - a_2 b_1)$$
$$= 0$$

How about $\vec{b} \times \vec{a}$?

How about $\vec{b} \times \vec{a}$? By the right hand rule



Some special cross products:

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What is $|\vec{a} \times \vec{b}|$?

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What is $|\vec{a} \times \vec{b}|$?

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \theta, \qquad \theta \text{ is the angle between } \vec{a} \text{ and } \vec{b}$$

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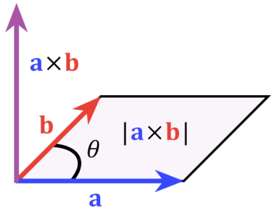
What is this? It is the area of the parallelogram with sides $\vec{a}$ and $\vec{b}$.



Proof:

$$
\begin{aligned}
&|\vec{a} \times \vec{b}|^2 = (a_2 b_3 - a_3 b_2)^2 + (a_3 b_1 - a_1 b_3)^2 + (a_1 b_2 - a_2 b_1)^2 \\
=&(a_2^2 b_3^2 - 2a_2 b_3 a_3 b_2 + a_3^2 b_2^2) + (a_3^2 b_1^2 - 2a_3 b_1 a_1 b_3 + a_1^2 b_3^2) + \\
&+ (a_1^2 b_2^2 - 2a_1 b_2 a_2 b_1 + a_2^2 b_1^2) \\
=&(a_2^2 b_3^2 + a_3^2 b_2^2 + a_3^2 b_1^2 + a_1^2 b_3^2 + a_1^2 b_2^2 + a_2^2 b_1^2) - \\
&- (2a_2 a_3 b_2 b_3 + 2a_1 a_3 b_1 b_3 + 2a_1 a_2 b_1 b_2) \\
=&(a_2^2 b_3^2 + a_3^2 b_2^2 + a_3^2 b_1^2 + a_1^2 b_3^2 + a_1^2 b_2^2 + a_2^2 b_1^2 + a_1^2 b_1^2 + a_2^2 b_2^2 + a_3^2 b_3^2) - \\
&- (2a_2 b_2 a_3 b_3 + 2a_1 b_1 a_3 b_3 + 2a_1 b_1 a_2 b_2 + a_1^2 b_1^2 + a_2^2 b_2^2 + a_3^2 b_3^2) \\
=&(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2) - (a_1 b_1 + a_2 b_2 + a_3 b_3)^2 \\
=&|\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 \\
=&|\vec{a}|^2 |\vec{b}|^2 - (|\vec{a}||\vec{b}| \cos\theta)^2 \\
=&|\vec{a}|^2 |\vec{b}|^2 (1 - \cos^2\theta) = |\vec{a}|^2 |\vec{b}|^2 \sin^2\theta \qquad \square
\end{aligned}
$$

$\vec{a} \times \vec{b} = \vec{0}$

$\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \sin\theta = 0$

$\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \sin\theta = 0 \Leftrightarrow \theta = 0 \text{ or } \pi$

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Back to the triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$. Geometric meaning?

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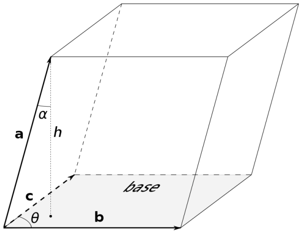
Back to the triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$. Geometric meaning? Its magnitude is the volume of the parallelepiped determined by $\vec{a}$, $\vec{b}$ and $\vec{c}$.



$V = \text{area of the base} \cdot \text{height}$

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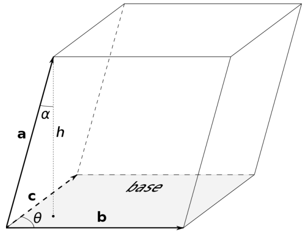

$$V = \text{area of the base} \cdot \text{height} = |\vec{b} \times \vec{c}||\vec{a}||\cos\alpha| = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$

where α is the angle between $\vec{a}$ and $\vec{b} \times \vec{c}$.

Physical Interpretation of Cross Product

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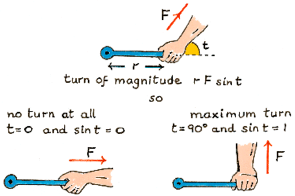
Tighten a bolt by applying a force $\vec{F}$ to a wrench at a point given by the position vector $\vec{r}$. This produces a turning effect. The **torque** $\vec{\tau}$ is defined as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Hence, $|\vec{\tau}| = |\vec{r}||\vec{F}| \sin\theta$, where θ is the angle between the position and the force vector.



http://en.wikipedia.org/wiki/Torque

Example 1 Determine whether the following expression is meaningful.

1.
$$(\vec{a} \cdot \vec{b}) \cdot \vec{c}$$

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1. $$(\vec{a} \cdot \vec{b}) \cdot \vec{c} \qquad \text{NO}$$

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$$\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 6\vec{i} + 3\vec{j} + 2\vec{k}$$

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(b) The area of $\triangle PQR$ is half of the area of the parallelogram determined by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. And the area of the parallelogram is

$$|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{36 + 9 + 4} = \sqrt{49} = 7$$

So

$$\text{Area}(\triangle PQR) = \frac{7}{2}$$