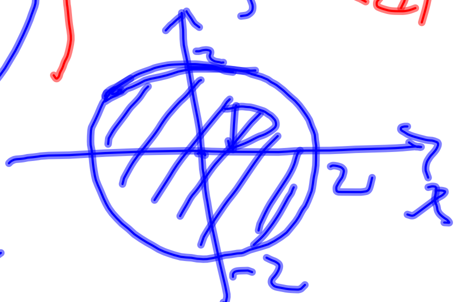
Ex2

$\sqrt{x^2+y^2} \le z \le 2 \Rightarrow r \le z \le 2$

$\begin{cases} -\sqrt{4-y^2} \le x \le \sqrt{4-y^2} \\ -2 \le y \le 2 \end{cases} \Rightarrow \begin{matrix} 0 \le r \le 2 \\ 0 \le \theta \le 2\pi \end{matrix}$



$x = \sqrt{4-y^2}$

$x^2 = 4-y^2 \Rightarrow x^2+y^2 = 4$

(r, θ, z)

$$\iiint = \int_0^{2\pi}\int_0^2\int_r^2 r\cos\theta \, z \, r \, dz\,dr\,d\theta$$

$$= \left[\int_0^{2\pi}\cos\theta \, d\theta\right] \int_0^2 r^2\left[\int_r^2 z\,dz\right]dr\,\cancel{d\theta}$$

0

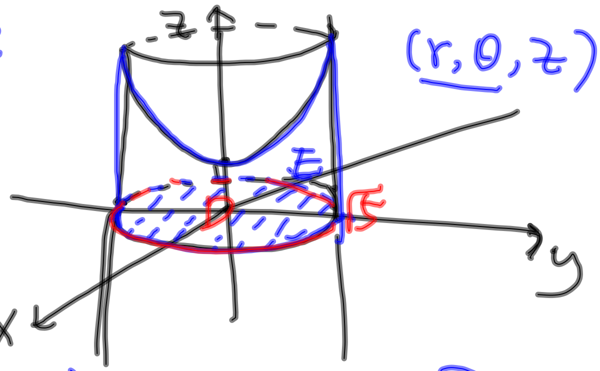
$$= \int_0^{2\pi}\cos\theta \int_0^2 r^2\left[\frac{1}{2}z^2\Big|_r^2\right]dr\,d\theta$$

$$= \int_0^{2\pi}\cos\theta \int_0^2 r^2\left[\frac{1}{2}\cdot 4 - \frac{1}{2}r^2\right]dr\,d\theta$$

$$= \int_0^{2\pi}\cos\theta \left[\int_0^2 r^2\left(2-\frac{1}{2}r^2\right)dr\right]d\theta$$

$$= 0$$

Ex3



(r, θ, z)

(r, θ) plane region D

$$\begin{cases} 0 \le r \le \sqrt{5} \\ 0 \le \theta \le 2\pi \end{cases} \qquad \begin{array}{l} 0 \le z \le 1 + x^2 + y^2 \\ 0 \le z \le 1 + r^2 \end{array}$$

$$\iiint_E e^z \, dV$$

$$= \int_0^{2\pi} \int_0^{\sqrt{5}} \int_0^{1+r^2} e^z r \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\sqrt{5}} r \left[e^z \Big|_0^{1+r^2} \right] dr \, d\theta$$

$$= \left[\int_0^{2\pi} d\theta \right] \left[\int_0^{\sqrt{5}} r \left[e^{1+r^2} - 1 \right] dr \right]$$

$$= 2\pi \left[\underbrace{\int_0^{\sqrt{5}} r e^{1+r^2} dr}_{} - \underbrace{\int_0^{\sqrt{5}} r \, dr}_{} \right]$$

let $u = 1 + r^2$
$\Rightarrow du = 2r \, dr$
$\Rightarrow r \, dr = \frac{1}{2} du$

$$\frac{1}{2} r^2 \Big|_0^{\sqrt{5}} = \frac{5}{2}$$

$$\frac{1}{2} \int_0^{\sqrt{5}} e^u \, du = \frac{1}{2} e^u \Big|_{r=0}^{\sqrt{5}}$$

$$= \frac{1}{2} e^{1+r^2} \Big|_0^{\sqrt{5}}$$

$$= \frac{1}{2} (e^6 - e)$$

$$\iiint = \cancel{2}\pi \left[\frac{1}{\cancel{2}} (e^6 - e) - \frac{5}{\cancel{2}} \right]$$

$$= \pi (e^6 - e - 5)$$

Ex1: ① $\begin{cases} x=1 \\ y=\sqrt{3} \\ z=2\sqrt{3} \end{cases} \longrightarrow (\rho,\theta,\phi)$

$\rho^2 = 1+3+4\cdot 3 = 4+12 = 16$

$\Rightarrow \rho = 4$

$z = \rho\cos\phi = 4\cos\phi = 2\sqrt{3}$

$\Rightarrow \cos\phi = \frac{\sqrt{3}}{2}$

$\Rightarrow \phi = \frac{\pi}{6}$

$x = \rho\sin\phi\cos\theta = 1$

$= 4\cdot\frac{1}{2}\cos\theta = 1$

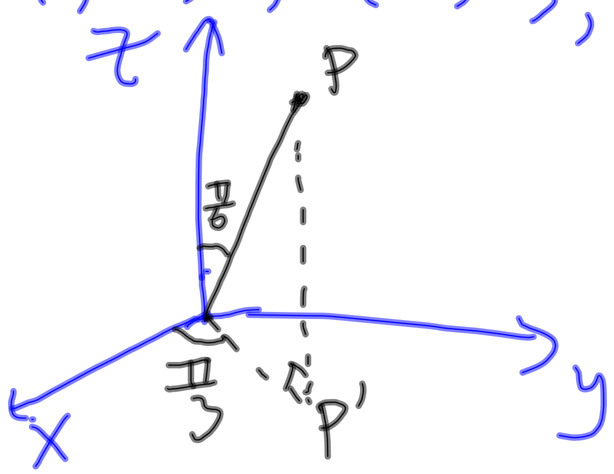
$\Rightarrow \cos\theta = \frac{1}{2}$ ✓

$y = \rho\sin\phi\sin\theta = \sqrt{3}$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$ ✓

$\Rightarrow \theta = \frac{\pi}{3}$

$(\rho,\theta,\phi) = (4, \frac{\pi}{3}, \frac{\pi}{6})$



② $\begin{cases} \rho = 2 \\ \theta = \frac{\pi}{3} \\ \phi = \frac{\pi}{4} \end{cases} \Rightarrow$ $z = \rho \cos\phi$

$= 2 \cdot \frac{\sqrt{2}}{2} = \boxed{\sqrt{2}}$

$x = \rho \sin\phi \cos\theta$

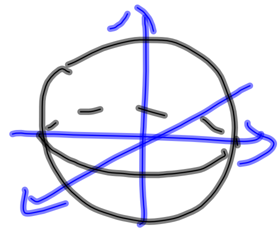
$= 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{2}}{2}}$

$y = \rho \sin\phi \sin\theta$

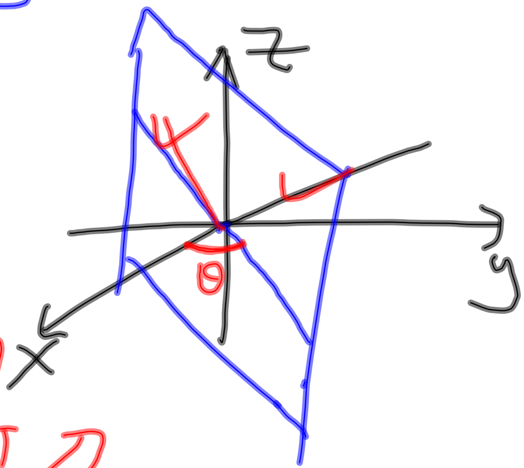
$= 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{6}}{2}}$

Ex2

① $\boxed{\rho = R}$



②

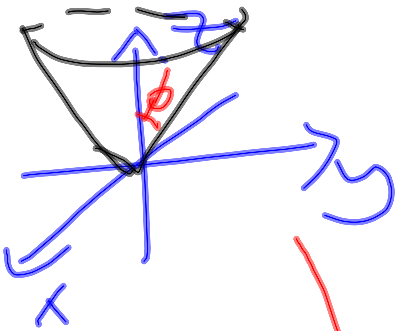


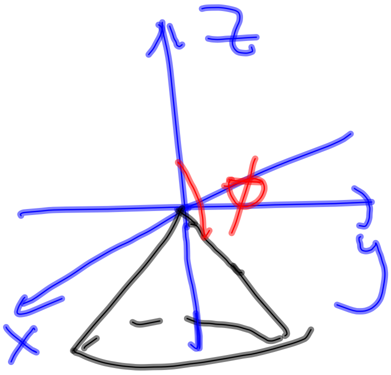
$$\tan\theta = \frac{y}{x} = k$$

$$\Rightarrow \boxed{\theta = \begin{cases}\arctan k \\ \arctan k + \pi\end{cases}}$$

$$\theta = \begin{cases} c \\ c+\pi \end{cases}$$

③





$\phi = \alpha,\ 0<\alpha<\frac{\pi}{2}$ | $\phi = \beta,\ \frac{\pi}{2}<\beta<\pi$

$\phi = 0 \Rightarrow z^+$ axis

$\phi = \frac{\pi}{2} \Rightarrow$ xy-plane

$\phi = \pi \Rightarrow z^-$-axis