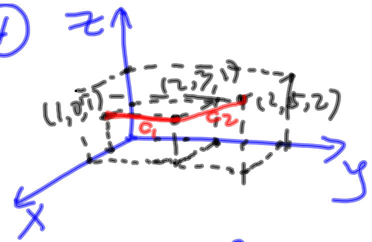
④



$$\int_C = \int_{C_1} + \int_{C_2}$$

C_1: $\vec{r}_0 = \langle 1,0,1\rangle$

$\vec{r}_1 = \langle 2,3,1\rangle$

$$\vec{r}(t) = (1-t)\vec{r}_0 + t\,\vec{r}_1$$

$$= (1-t)\langle 1,0,1\rangle + t\langle 2,3,1\rangle$$

$$= \langle 1-t, 0, 1-t\rangle + \langle 2t, 3t, t\rangle$$

$$= \langle 1+t, 3t, 1\rangle$$

$$\Rightarrow \boxed{\begin{cases} x = 1+t \\ y = 3t \\ z = 1 \end{cases}} \Rightarrow \begin{cases} dx = dt \\ dy = 3dt \\ dz = 0 \end{cases}$$

C_2: $\vec{r}_1 = \langle 2,3,1\rangle$, $\vec{r}_2 = \langle 2,5,2\rangle$

$$\vec{s}(u) = (1-u)\vec{r}_1 + u\,\vec{r}_2$$

$$= \langle 2-2u, 3-3u, 1-u\rangle$$

$$+ \langle 2u, 5u, 2u\rangle$$

$$= \langle 2, 3+2u, 1+u\rangle$$

$$\boxed{\begin{cases} x(u) = 2 \\ y(u) = 3+2u \\ z(u) = 1+u \end{cases} \Rightarrow \begin{matrix} dx = 0 \\ dy = 2du \\ dz = du \end{matrix}}$$

$$\int_{C_1} (x+yz)dx + 2x\,dy + xyz\,dz$$

$$= \int_0^1 (1+t+3t)dt + 2(1+t)3dt$$

$$= \int_0^1 (1+4t+6+6t)dt$$

$$= \int_0^1 (7+10t)dt$$

$$= 7t+5t^2\Big|_0^1 = 7+5 = 12$$

$$\int_{C_2} 0 + 2\cdot 2\cdot 2du +$$

$$2(3+2u)(1+u)\,du$$

$$= \int_0^1 \left(8 + 2(2u^2+5u+3)\right)du$$

$$= \int_0^1 (8+4u^2+10u+6)du$$

$$= \frac{4}{3}u^3 + 5u^2 + 14u\Big|_0^1$$

$$= \frac{4}{3} + 5 + 14 = \frac{4}{3} + 19$$

$$= \frac{61}{3}$$

$$\begin{array}{r} 19 \\ \times\ 3 \\ \hline 57 \end{array}$$

$$\int_C = 12 + \frac{61}{3}$$

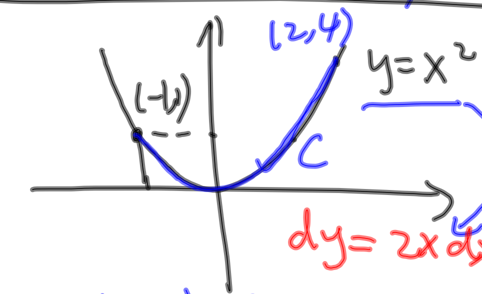
$$= \boxed{\frac{97}{3}}$$

$$\begin{array}{r} 36 \\ 61 \\ \hline \end{array}$$

Find the work done by the force field

$\vec{F}(x,y) = x\sin y\,\vec{i} + y\,\vec{j}$

on a particle that moves along the parabola $y = x^2$ from $(-1,1)$ to $(2,4)$



$dy = 2x\,dx$

$$\int_C \vec{F}\cdot d\vec{r} = \int P\,dx + Q\,dy$$

$$= \int x\sin y\,dx + y\,dy$$

$$= \int_{-1}^{2} x\sin(x^2)\,dx + x^2 \cdot 2x\,dx$$

$$= \int_{-1}^{2} \left(x\sin(x^2) + 2x^3\right)dx$$

§17.3

Ex 1 $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ ✓

$$\frac{\partial(ye^x+\sin y)}{\partial y} = e^x+\cos y$$

$$\frac{\partial(e^x+x\cos y)}{\partial x} = e^x+\cos y$$

$\nabla f = \vec{F}$?

$$\vec{F} = (ye^x+\sin y)\vec{i} + (e^x+x\cos y)\vec{j}$$

If $\nabla f = \vec{F}$

$$= \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j}$$

$$\Rightarrow \begin{cases} \frac{\partial f}{\partial x} = ye^x+\sin y \\ \frac{\partial f}{\partial y} = e^x+x\cos y \end{cases}$$

$$f(x,y) = \int (ye^x+\sin y)\,dx$$

$$= ye^x + x\sin y + \boxed{G(y)}$$

$$\frac{\partial f}{\partial y} = e^x+x\cos y+G'(y)$$

$$= e^x+x\cos y$$

$$\Rightarrow G'(y)=0 \Rightarrow G(y)=C$$

$$\Rightarrow f(x,y) = ye^x+x\sin y+C$$

$$\nabla f = \vec{F}$$

② $\nabla f = f_x \vec{i} + f_y \vec{j} + f_z \vec{k}$
$= \vec{F}$

$\Rightarrow \begin{cases} \dfrac{\partial f}{\partial x} = yz \\ \dfrac{\partial f}{\partial y} = xz \\ \dfrac{\partial f}{\partial z} = xy + 2z \end{cases}$

$\int f_x \, dx = \int yz \, dx$

$\Rightarrow f(x,y,z) = xyz + G(y,z)$

$\Rightarrow \dfrac{\partial f}{\partial y} = xz + \dfrac{\partial G}{\partial y} = xz$

$\Rightarrow \dfrac{\partial G}{\partial y} = 0 \Rightarrow G(y,z) = \underline{G(z)}$

$\dfrac{\partial f}{\partial z} = xy + G'(z) = xy + 2z$

$\Rightarrow G'(z) = 2z$

$\Rightarrow G(z) = z^2 + C$

$\Rightarrow \boxed{f(x,y,z) = xyz + z^2 + C}$

$\int_C \vec{F} \cdot d\vec{r}$ $\quad$ $C: (1,0,-2) \to (4,6,3)$

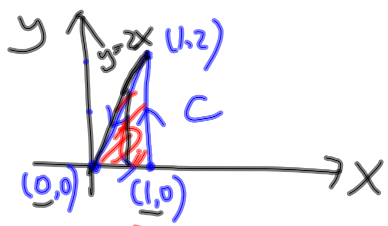
$= \int_C \nabla f \cdot d\vec{r}$ $\quad$ Fundamental Thm

$= f(4,6,3) - f(1,0,-2)$

$= \boxed{}$

§17.4

Ex 1



$$\oint_C xy\,dx + x^2y^3\,dy$$

$$= \iint_D \left[\frac{\partial(x^2y^3)}{\partial x} - \frac{\partial(xy)}{\partial y}\right] dA$$

$$= \iint_D (2xy^3 - x)\,dA$$

$$= \int_0^1 \int_0^{2x} (2xy^3 - x)\,dy\,dx$$

$$= \int_0^1 \left(\frac{2x}{4}y^4 - xy\right)\Big|_0^{2x} dx$$

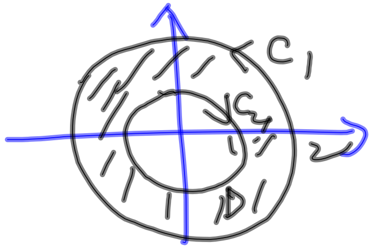
$$= \int_0^1 \left(\frac{x}{2}\cdot(2x)^4 - x\cdot 2x\right) dx$$

$$= \int_0^1 (8x^5 - 2x^2)\,dx$$

$$= \frac{8}{6}x^6 - \frac{2}{3}x^3\Big|_0^1$$

$$= \frac{4}{3} - \frac{2}{3} = \boxed{\frac{2}{3}}$$

Ex 2



$$\oint_C y^3dx - x^3dy$$

$$= \iint_D \left(\frac{\partial(-x^3)}{\partial x} - \frac{\partial(y^3)}{\partial y} \right) dA$$

$$= \iint_D (-3x^2 - 3y^2)\, dA$$

$$= \int_0^{2\pi}\int_1^2 (-3r^2)\, r\, dr\, d\theta$$

$$= \left(\int_0^{2\pi} d\theta\right)\left[\int_1^2 -3r^3\, dr\right]$$

$$= 2\pi \left[-\frac{3}{4} r^4 \Big|_1^2 \right)$$

$$= 2\pi \left(-\frac{3}{4}\right)(16-1)$$

$$= \boxed{-\frac{45}{2}\pi}$$