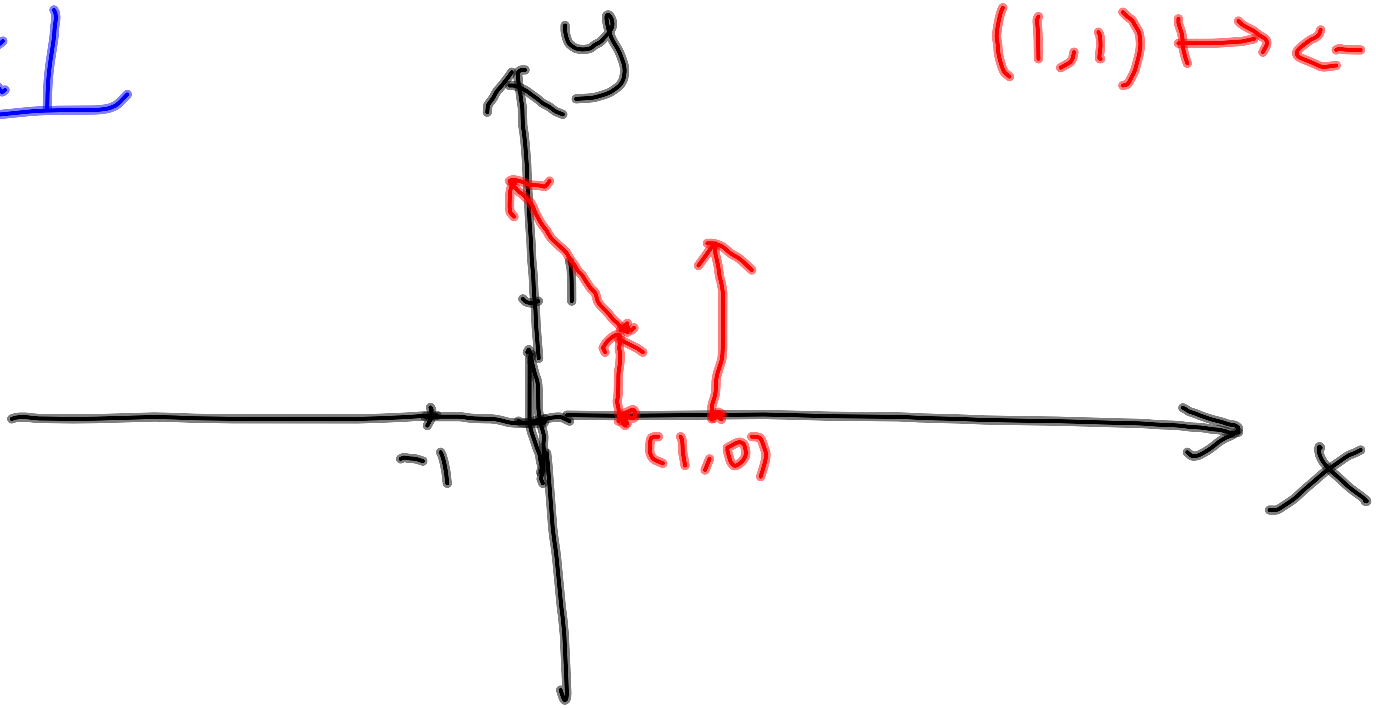
Ex 1

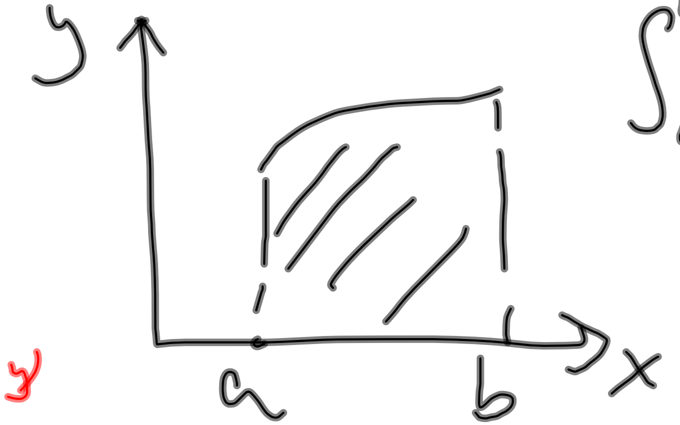
$(1,1) \mapsto \langle -1, 1\rangle$



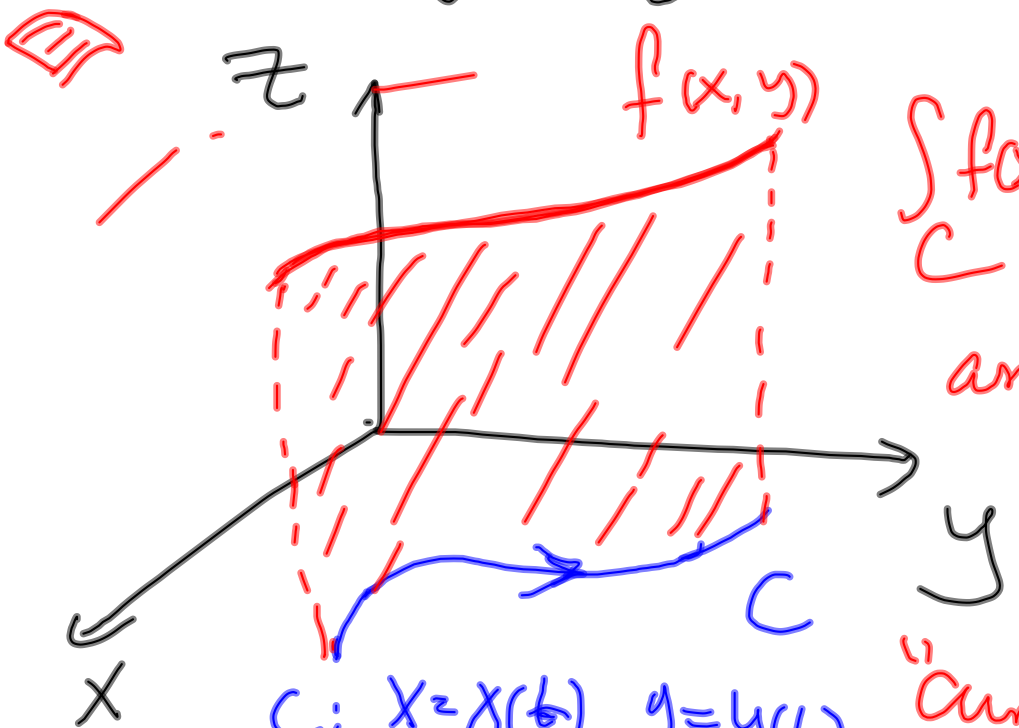
$$\vec{F}(x,y) = -y\,\vec{i} + x\,\vec{j}$$

pt $(x,y) \longmapsto \langle -y, x\rangle$

$$\int_a^b f(x)\,dx$$



$z = f(x,y)$



$$\int_C f(x,y)\,ds$$

" area of "curtain"

$C:\ x = x(t),\ y = y(t),\ a \le t \le b$

§17.2

Ex1

$C: \begin{cases} x = 4\cos t \\ y = 4\sin t \end{cases}$

$-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

$\frac{dx}{dt} = -4\sin t$

$\frac{dy}{dt} = 4\cos t$

$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$= \sqrt{16\sin^2 t + 16\cos^2 t}$

$= 4$

$\int_C x y^4 \, ds$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\cos t \,(4\sin t)^4 \cdot 4 \, dt$

$= 16 \cdot 4^4 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t \,(\sin t)^4 \, dt$

let $u = \sin t \Rightarrow du = \cos t \, dt$

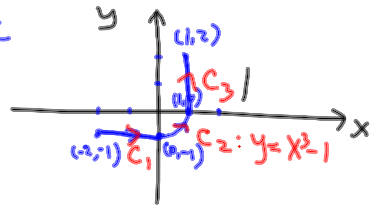
$= 16 \cdot 256 \int_{-1}^{1} u^4 \, du$

$= 16 \cdot 256 \left[\frac{1}{5} u^5\right]_{-1}^{1}$

$= \frac{16 \cdot 256}{5} [1 + 1]$

$= \frac{16 \cdot 256 \cdot 2}{5} = \frac{256 \cdot 32}{5}$

Ex 2



$$\int_C 4x^3\,ds = \int_{C_1} 4x^3\,ds + \int_{C_2} 4x^3\,ds + \int_{C_3} 4x^3\,ds$$

① $C_1: \begin{cases} y=-1 \\ x=t, \ -2\le t\le 0 \end{cases}$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$= \sqrt{1+0}\,dt = dt$$

$$\int_{C_1} 4x^3\,ds = \int_{-2}^{0} 4\cdot t^3\,dt$$

$$= t^4\Big|_{-2}^{0}$$

$$= 0-(-2)^4 = -16$$

② $C_2: \begin{cases} y=t^3-1 \\ x=t, \ 0\le t\le 1 \end{cases}$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$= \sqrt{1+(3t^2)^2}\,dt$$

$$= \sqrt{1+9t^4}\,dt$$

$$\int_{C_2} 4x^3\,ds = \int_0^1 4t^3\sqrt{1+9t^4}\,dt$$

Let $u = 1+9t^4$

$\Rightarrow du = 9\cdot 4t^3\,dt$

$\Rightarrow 4t^3\,dt = \frac{1}{9}\,du$

$$\int_{C_2} 4x^3 ds = \int_0^1 \frac{1}{9}\sqrt{u}\, du$$

$$= \frac{1}{9}\frac{2}{3} u^{\frac{3}{2}} \Big|_{t=0}^{1}$$

$$= \frac{2}{27}\left(1+9t^4\right)^{\frac{3}{2}} \Big|_0^1$$

$$= \frac{2}{27}\left[(1+9)^{\frac{3}{2}} - 1\right]$$

$$= \frac{2}{27}\left[10^{\frac{3}{2}} - 1\right]$$

③ C_3 : $\begin{cases} x = 1 \\ y = t, \ 0 \le t \le 2 \end{cases}$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \sqrt{0+1}\, dt = dt$$

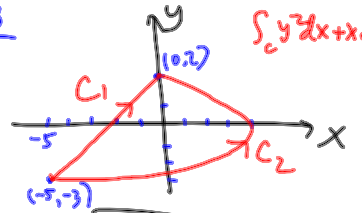
$$\int_{C_3} 4x^3 ds = \int_0^2 4 \cdot dt = 4 \cdot 2$$

$$= 8$$

$$\int_C 4x^3 ds = -16 + \frac{2}{27}\left[10^{\frac{3}{2}} - 1\right] + 8$$

$$= -8 + \frac{2}{27}\left[10^{\frac{3}{2}} - 1\right]$$

Ex3



$\int_C y^2 dx + x\,dy$

(a) C_1: (x_0, y_0), (x_1, y_1)

$$\boxed{\begin{array}{l} \vec{r}_0 = \langle x_0, y_0 \rangle,\ \vec{r}_1 = \langle x_1, y_1 \rangle \\ \underline{\vec{r}(t) = (1-t)\vec{r}_0 + t\,\vec{r}_1}, \\ \quad 0 \le t \le 1 \end{array}}$$

$\vec{r}_0 = \langle -5, -3 \rangle$, $\vec{r}_1 = \langle 0, 2 \rangle$

$\vec{r}(t) = (1-t)\langle -5, -3 \rangle + t\langle 0, 2 \rangle$

$= \langle -5(1-t), -3(1-t) \rangle$

$+ \langle 0, 2t \rangle$

$= \langle -5+5t, -3+3t+2t \rangle$

$= \langle 5t-5, 5t-3 \rangle$

$\Rightarrow \underline{x(t) = 5t-5}$, $y(t) = 5t-3$,

$dx = 5\,dt \qquad 0 \le t \le 1$

$\Rightarrow \int_{C_1} y^2 dx + x\,dy$

$= \int_0^1 (5t-3)^2 \cdot 5\,dt + (5t-5) \cdot 5\,dt$

$= \int_0^1 5\left[(5t-3)^2 + (5t-5)\right] dt$

$= \int_0^1 5\,[25t^2 - 30t + 9$

$+ 5t - 5]\,dt$

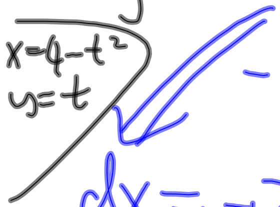
$$= 5\int_0^1 \left(25t^2 - 25t + 4\right) dt$$

$$= 5\left[\frac{25}{3}t^3 - \frac{25}{2}t^2 + 4t\right]_0^1$$

$$= 5\left[\frac{25}{3} - \frac{25}{2} + 4\right]$$

$$= \boxed{-\frac{5}{6}}$$

Along C_2: $x=4-y^2$, $-3\le y\le 2$



$\underline{dx=-2y\,dy}$, $dy=dy$

$$\int_{C_2} y^2\,dx + x\,dy$$

$$=\int_{-3}^{2} y^2(-2y\,dy)+(4-y^2)\,dy$$

$$=\int_{-3}^{2}\left(-2y^3+4-y^2\right)dy$$

$$=-\frac{2}{4}y^4+4y-\frac{1}{3}y^3\Big|_{-3}^{2}$$

$$=-\frac{1}{2}(16-81)+4(2+3)-\frac{1}{3}(8+27)$$

$$=\boxed{40\frac{5}{6}}$$