Ex 1

$$\int_0^1\int_1^2\int_2^3 8xyz\,dx\,dy\,dz$$

$$= 8\int_0^1 z \int_1^2 y\left[\int_2^3 x\,dx\right]dy\,dz$$

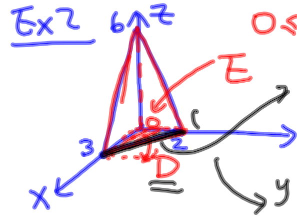
$$= 8\left[\int_0^1 z\,dz\right]\left[\int_1^2 y\,dy\right]\cdot\left[\int_2^3 x\,dx\right]$$

$$= 8\left[\frac{1}{2} z^2\Big|_0^1\right]\left[\frac{1}{2} y^2\Big|_1^2\right]\cdot\left[\frac{1}{2} x^2\Big|_2^3\right]$$

$$= \cancel{8}\cdot\frac{1}{\cancel{2}}\cdot\frac{1}{\cancel{2}}(4-1)\cdot\frac{1}{\cancel{2}}(9-4)$$

$$= 3\cdot 5 = \boxed{15}$$

Ex 2

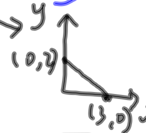


$0 \le z \le 6-2x-3y$

$2x+3y+z=6$

$z=0$

$\Longrightarrow 2x+3y=6$



$(x,y)\in D$

$0 \le x \le 3$

$0 \le y \le -\frac{2}{3}x+2$

$\frac{0-2}{3-0} = \frac{y-0}{x-3}$

$\Rightarrow y = -\frac{2}{3}(x-3)$

$= -\frac{2}{3}+2$

$$\iiint_E 2x\,dV$$

$$= \int_0^3 \int_0^{-\frac{2}{3}x+2} \int_0^{6-2x-3y} 2x\,dz\,dy\,dx$$

$$= \int_0^3 2x\left[\int_0^{-\frac{2}{3}x+2} (6-2x-3y)\,dy\right]dx$$

$$= \int_0^3 2x\left[6y-2xy-\frac{3}{2}y^2\right]_0^{-\frac{2}{3}x+2} dx$$

$$= \int_0^3 2x\left[6\left(-\frac{2}{3}x+2\right)-2x\left(-\frac{2}{3}x+2\right) - \frac{3}{2}\left(-\frac{2}{3}x+2\right)^2\right]dx$$

$$= \int_0^3 2x\left[-4x+12+\frac{4}{3}x^2-4x - \frac{3}{2}\left(\frac{4}{9}x^2-\frac{8}{3}x+4\right)\right]dx$$

$$= \int_0^3 2x\left[-8x+12+\frac{4}{3}x^2 - \frac{2}{3}x^2+4x-6\right]dx$$

$$= \int_0^3 2x\left[-4x+6+\frac{2}{3}x^2\right]dx$$

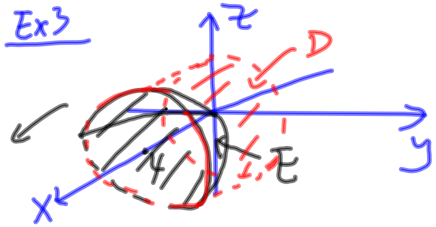
$$= \int_0^3 -8x^2+12x+\frac{4}{3}x^3\,dx$$

$$= -\frac{8}{3}x^3+6x^2+\frac{1}{3}x^4 \Big|_0^3$$

$$= -\frac{8}{\cancel{3}}\cancel{27}^{\,9}+6\cdot 9+\frac{1}{\cancel{3}}\cdot\cancel{81}\,27$$

$$= -72+54+27 = \boxed{9}$$

Ex3



$4y^2+4z^2 \le x \le \underline{4}$

intersection of $x=4y^2+4z^2$
& $x=4$

$\Rightarrow 4y^2+4z^2=4$
$\Leftrightarrow y^2+z^2=1 \Rightarrow \partial D$

$D=\{(y,z) \mid y^2+z^2\le 1\}$

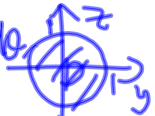
$$\iiint_E x\,dV = \iint_D \left[\int_{4y^2+4z^2}^{4} x\,dx\right] dA$$

$$= \iint_D \tfrac{1}{2}x^2\Big|_{4y^2+4z^2}^{4}\, dA$$

$$= \iint_D \left[\tfrac{1}{2}\cdot 16 - \tfrac{1}{2}\cdot 16(y^2+z^2)^2\right] dA$$

$$= 8 \iint_D \left[1-(y^2+z^2)^2\right] dA$$

$$= 8\int_0^{2\pi}\int_0^1 (1-r^4)\,r\,dr\,d\theta$$



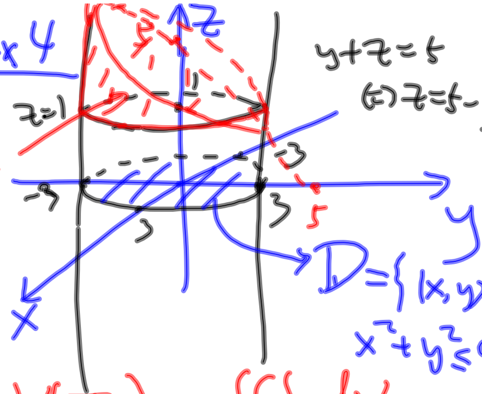
$$= 8\cdot 2\pi \int_0^1 (r-r^5)\,dr$$

$$= 16\pi\left[\tfrac{1}{2}r^2-\tfrac{1}{6}r^6\right]_0^1$$

$$= 16\pi\left[\tfrac{1}{2}-\tfrac{1}{6}\right]$$

$$= 16\pi\cdot\tfrac{2}{6} = \boxed{\tfrac{16}{3}\pi}$$

Ex 4



$y+z=5 \Leftrightarrow z=5-y$

$D=\{(x,y) \mid x^2+y^2 \le 9\}$

$$V(E) = \iiint_E dV$$

$$1 \le z \le 5-y$$

$$V = \iint_D \left[\int_1^{5-y} dz\right] dA$$

$$= \iint_D (5-y-1)\, dA$$

$$= \int_0^{2\pi}\int_0^3 (4 - r\sin\theta)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \left[2r^2 - \frac{1}{3}r^3 \sin\theta\right]_0^3 d\theta$$

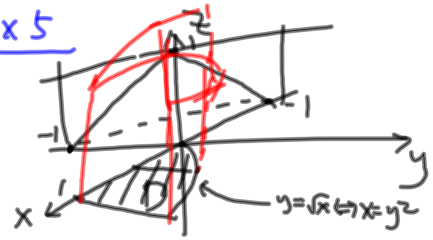
$$= \int_0^{2\pi} \left[2\cdot 9 - \frac{1}{3}\cdot 3^3 \sin\theta\right] d\theta$$

$$= \int_0^{2\pi} (18 - 9\sin\theta)\, d\theta$$

$$= 18\theta\Big|_0^{2\pi} + \cancel{9\cos\theta\Big|_0^{2\pi}}$$

$$= \boxed{36\pi}$$

Ex 5



$$m = \iiint_E \rho \, dV$$

$$0 \le z \le 1+x+y$$

On D,

$$0 \le y \le \sqrt{x}$$
$$0 \le x \le 1$$

$$m = \rho \int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} dz\,dy\,dx$$

$$= \rho \int_0^1 \int_0^{\sqrt{x}} (1+x+y)\,dy\,dx$$

$$= \rho \int_0^1 \left[y + xy + \tfrac{1}{2}y^2\right]_0^{\sqrt{x}} dx$$

$$= \rho \int_0^1 \left(\sqrt{x} + \underbrace{x\sqrt{x}}_{x^{\frac{3}{2}}} + \tfrac{1}{2}x\right) dx$$

$$= \rho \left[\frac{1}{1+\frac{1}{2}} x^{\frac{3}{2}} + \frac{1}{1+\frac{3}{2}} x^{\frac{5}{2}} + \frac{1}{4}x^2\right]_0^1$$

$$= \rho \left[\frac{2}{3} + \frac{2}{5} + \frac{1}{4}\right]$$

$$= \rho \, \frac{40+24+15}{60} = \frac{79}{60}\rho$$

$$\begin{array}{r} 24 \\ 15 \\ \hline 39 \end{array}$$

$$M_{xy} = \iiint_E z\rho \, dV$$
$$M_{xz} = \iiint_E y\rho \, dV$$
$$M_{yz} = \iiint_E x\rho \, dV$$

$(\bar{x}, \bar{y}, \bar{z})$