Math 114-004, Fall 2009

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Department of Mathematics
University of Pennsylvania

October 13, 2009

Midterm I Review

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Covered:

1. Linear Differential Equations:

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1. Linear Differential Equations: $y' + P(x)y = Q(x)$:

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3. 3D Coordinate Systems:

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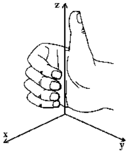
   - 

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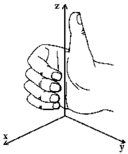
   - 

   - distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$

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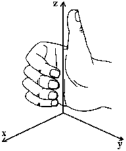
   - 

   - distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ $|P_1P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$

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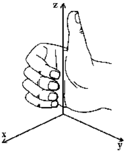
   - 

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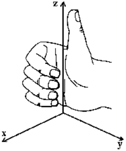
   - 

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   - equation of a sphere centered at (a, b, c) with radius r: $(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$

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   - $|\vec{a} \cdot (\vec{b} \times \vec{c})|$:

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   - $|\vec{a} \cdot (\vec{b} \times \vec{c})|$: volume of the parallelepiped

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   - vector equation: $\vec{r} = \vec{r}_0 + t\vec{v}$
   - parametric equations: $x = x_0 + at, y = y_0 + bt, z = z_0 + ct$
   - symmetric equations: $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$
8. Equations of Planes: given a point $P_0 = (x_0, y_0, z_0)$ and a normal vector $\vec{n} =< a, b, c >$
   - vector equation: $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$
   - scalar equation: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ or $ax + by + cz + d = 0$
   - two planes are parallel if and only if $\vec{n}_1 = c\vec{n}_2$
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$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

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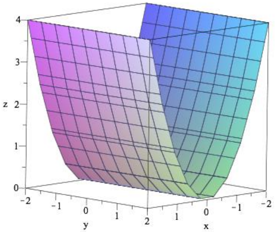

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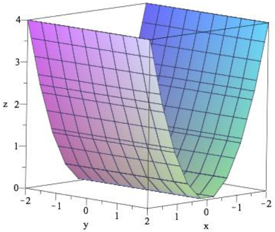

- $x^2 + y^2 = 1$

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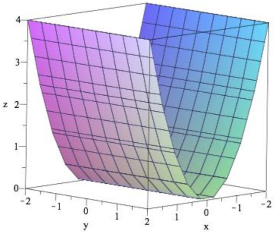
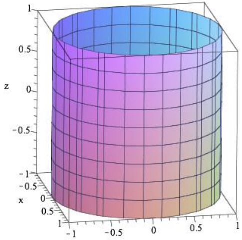


- $x^2 + y^2 = 1$



- $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$

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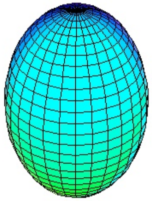

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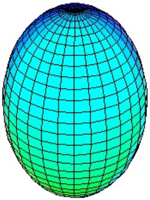

- $z = 4x^2 + y^2$

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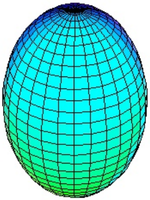

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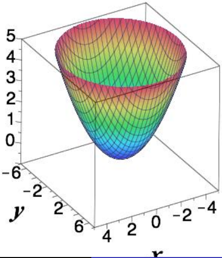

- $z = y^2 - x^2$

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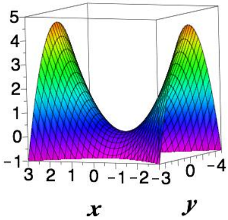

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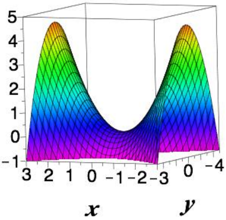

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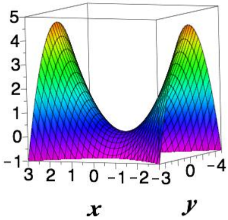

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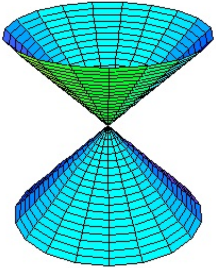

- $z^2 = x^2 + \frac{y^2}{4} - 1$

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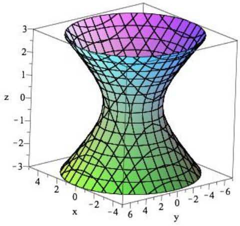

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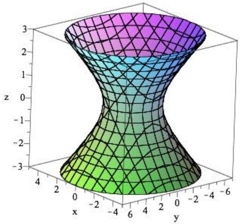
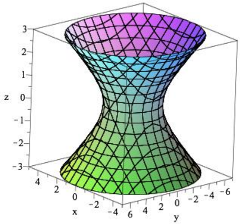


- $z^2 = x^2 + \frac{y^2}{4} + 1$

- $z^2 = x^2 + \frac{y^2}{4} - 1$



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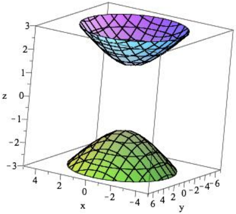

10. Vector Functions:

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    - tangent vector at t_0:

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    - derivative $\vec{r'}(t) = < f'(t), g'(t), h'(t) >$
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    - tangent vector at t_0: $\vec{r'}(t_0)$;

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   - tangent vector at t_0: $\vec{r'}(t_0)$; tangent line at t_0:

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    - tangent vector at t_0: $\vec{r'}(t_0)$; tangent line at t_0: the point $P_0 = (f(t_0), g(t_0), h(t_0))$ and the direction vector $\vec{r'}(t_0)$; unit tangent vector: $\vec{T}(t) = \frac{\vec{r'}(t)}{|\vec{r'}(t)|}$
12. Motion in Space:

10. Vector Functions: $\vec{r}(t) =< f(t), g(t), h(t) >$
    - limit $\lim\limits_{t \to t_0} \vec{r}(t) =< \lim\limits_{t \to t_0} f(t), \lim\limits_{t \to t_0} g(t), \lim\limits_{t \to t_0} h(t) >$
    - derivative $\vec{r'}(t) =< f'(t), g'(t), h'(t) >$
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    - tangent vector at t_0: $\vec{r'}(t_0)$; tangent line at t_0: the point $P_0 = (f(t_0), g(t_0), h(t_0))$ and the direction vector $\vec{r'}(t_0)$; unit tangent vector: $\vec{T}(t) = \frac{\vec{r'}(t)}{|\vec{r'}(t)|}$
12. Motion in Space: curve $\vec{r}(t)$

10. Vector Functions: $\vec{r}(t) =< f(t), g(t), h(t) >$
   - limit $\lim_{t \to t_0} \vec{r}(t) =< \lim_{t \to t_0} f(t), \lim_{t \to t_0} g(t), \lim_{t \to t_0} h(t) >$
   - derivative $\vec{r'}(t) =< f'(t), g'(t), h'(t) >$
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11. Space curve: a vector function $\vec{r}(t) =< f(t), g(t), h(t) >$ generates a space curve $x = f(t), y = g(t), z = h(t)$
   - tangent vector at t_0: $\vec{r'}(t_0)$; tangent line at t_0: the point $P_0 = (f(t_0), g(t_0), h(t_0))$ and the direction vector $\vec{r'}(t_0)$; unit tangent vector: $\vec{T}(t) = \frac{\vec{r'}(t)}{|\vec{r'}(t)|}$
12. Motion in Space: curve $\vec{r}(t)$
   - velocity vector:

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    - limit $\lim_{t\to t_0} \vec{r}(t) =< \lim_{t\to t_0} f(t), \lim_{t\to t_0} g(t), \lim_{t\to t_0} h(t) >$
    - derivative $\vec{r'}(t) =< f'(t), g'(t), h'(t) >$
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11. Space curve: a vector function $\vec{r}(t) =< f(t), g(t), h(t) >$ generates a space curve $x = f(t), y = g(t), z = h(t)$
    - tangent vector at t_0: $\vec{r'}(t_0)$; tangent line at t_0: the point $P_0 = (f(t_0), g(t_0), h(t_0))$ and the direction vector $\vec{r'}(t_0)$; unit tangent vector: $\vec{T}(t) = \frac{\vec{r'}(t)}{|\vec{r'}(t)|}$
12. Motion in Space: curve $\vec{r}(t)$
    - velocity vector: $\vec{v}(t) = \vec{r'}(t)$

10. Vector Functions: $\vec{r}(t) =< f(t), g(t), h(t) >$
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   - tangent vector at t_0: $\vec{r'}(t_0)$; tangent line at t_0: the point $P_0 = (f(t_0), g(t_0), h(t_0))$ and the direction vector $\vec{r'}(t_0)$; unit tangent vector: $\vec{T}(t) = \frac{\vec{r'}(t)}{|\vec{r'}(t)|}$
12. Motion in Space: curve $\vec{r}(t)$
   - velocity vector: $\vec{v}(t) = \vec{r'}(t)$
   - acceleration vector:

10. Vector Functions: $\vec{r}(t) =< f(t), g(t), h(t) >$
   - limit $\lim_{t\to t_0} \vec{r}(t) =< \lim_{t\to t_0} f(t), \lim_{t\to t_0} g(t), \lim_{t\to t_0} h(t) >$
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   - velocity vector: $\vec{v}(t) = \vec{r'}(t)$
   - acceleration vector: $\vec{a}(t) = \vec{v'}(t)$

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12. Motion in Space: curve $\vec{r}(t)$
   - velocity vector: $\vec{v}(t) = \vec{r'}(t)$
   - acceleration vector: $\vec{a}(t) = \vec{v'}(t)$
   - Newton's second law of motion:

10. Vector Functions: $\vec{r}(t) =< f(t), g(t), h(t) >$
    - limit $\lim_{t\to t_0} \vec{r}(t) =< \lim_{t\to t_0} f(t), \lim_{t\to t_0} g(t), \lim_{t\to t_0} h(t) >$
    - derivative $\vec{r'}(t) =< f'(t), g'(t), h'(t) >$
    - integration $\int_a^b \vec{r}(t)dt =< \int_a^b f(t)dt, \int_a^b g(t)dt, \int_a^b h(t)dt >$
11. Space curve: a vector function $\vec{r}(t) =< f(t), g(t), h(t) >$ generates a space curve $x = f(t), y = g(t), z = h(t)$
    - tangent vector at t_0: $\vec{r'}(t_0)$; tangent line at t_0: the point $P_0 = (f(t_0), g(t_0), h(t_0))$ and the direction vector $\vec{r'}(t_0)$; unit tangent vector: $\vec{T}(t) = \frac{\vec{r'}(t)}{|\vec{r'}(t)|}$
12. Motion in Space: curve $\vec{r}(t)$
    - velocity vector: $\vec{v}(t) = \vec{r'}(t)$
    - acceleration vector: $\vec{a}(t) = \vec{v'}(t)$
    - Newton's second law of motion: $\vec{F}(t) = m\vec{a}(t)$

Example 1 Solve the differential equation $xy' = y + x^2 \sin x$ with initial condition $y(\pi) = 0$.

Solution: It is a linear differential equation. Write it in the standard way:

$$y' - \frac{1}{x}y = x \sin x$$

So the integrating factor is $I(x) = e^{\int -1/x dx} = 1/|x|$. We only need to multiply $1/x$ to both sides of the DE, because if we multiply $-1/x$, the -1 can be cancelled out on both sides of the differential equation.

$$(I(x)y)' = \sin x \Rightarrow I(x)y = \int \sin x dx = -\cos x + C$$

$$\Rightarrow y(x) = -x \cos x + Cx$$

By the initial condition $y(\pi) = 0$, we can get $C = -1$. Hence, $y(x) = -x \cos x - x$.

Example 2 Let $\vec{v} =< 0, 7, 0 >$ and $\vec{u}$ be a vector of length 5 which starts at the origin and lies in the xy-plane. What is the maximum value of the length of $\vec{u} \times \vec{v}$?

Solution: Since $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin\theta$, when $\sin\theta = 1$, $|\vec{u} \times \vec{v}|$ gets the maximum 35.

Example 3 What conditions should x and y satisfy so that $\vec{a} =< x + y, 1, y >$ and $\vec{b} =< 1, x - y, -1 >$ are perpendicular to each other?

Solution: Two vectors are perpendicular to each other if and only if their dot product is easy. And

$$\vec{a} \cdot \vec{b} = x + y + x - y - y = 2x - y = 0$$

So, if x and y satisfy $2x - y = 0$, $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

Example 4 Determine whether the following statements are true or false.

1. For any 3-dimensional vectors $\vec{u}$ and $\vec{v}$, $\vec{u} \times \vec{v} = \vec{v} \times \vec{u}$.
2. The cross product of two unit vectors is a unit vector.
3. The vectors $< 1, 0, 1 >$ and $< 2, 0, 2 >$ are parallel.
4. The velocity vector of a curve in three dimensional space is always perpendicular to the acceleration vector.
5. The planes are parallel if the cross product of their normal vectors is 0.

Answer: 1. F 2. F 3. T 4. F 5. T

Example 5 Find the equation of the plane through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$ and perpendicular to the plane $x + y - 2z = 1$.

Solution: To decide the equation of a plane, we need a point which the plane passes through and a normal vector.

(1) To find a normal vector $\vec{n}$ to the plane, we need to analyze the two given conditions.

- Since the intersection line L is in the plane, $\vec{n}$ is perpendicular to L. If $\vec{v}$ is a direction vector to L, $\vec{n} \perp \vec{v}$.
- The plane is perpendicular to $x + y - 2z = 1$. If $\vec{n_0}$ is the normal vector to $x + y - 2z = 1$, then $\vec{n} \perp \vec{n_0}$.

Hence, we may let $\vec{n} = \vec{v} \times \vec{n_0}$. $\vec{n_0}$ is easy to decide: $\vec{n_0} =< 1, 1, -2 >$. How to find $\vec{v}$?

Since L is in both $x - z = 1$ and $y + 2z = 3$, it is perpendicular to the normal vectors of these two planes. Let $\vec{n_1}$ be the normal vector to $x - z = 1$. So $\vec{n_1} =< 1, 0, -1 >$. Let $\vec{n_2}$ be the normal vector to $y + 2z = 3$. So $\vec{n_2} =< 0, 1, 2 >$. The directional vector $\vec{v}$ to L is perpendicular to both $\vec{n_1}$ and $\vec{n_2}$. We may let

$$\vec{v} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = \vec{i} - 2\vec{j} + \vec{k}$$

So the normal vector $\vec{n}$ to the plane we want to find is

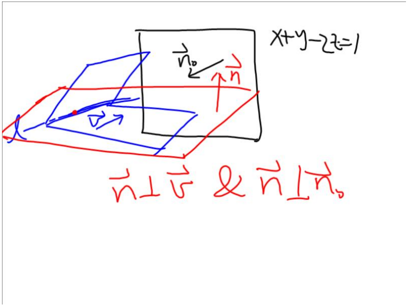
$$\vec{n} = \vec{v} \times \vec{n_0} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix} = 3\vec{i} + 3\vec{j} + 3\vec{k}$$

(2) To find a point in the plane, we only need to find a point from the line L, since L is in the plane. Say we choose $z = 0$, then $x = 1$ and $y = 3$. So the point $(1, 3, 0)$ is in the plane.

The equation to the plane is

$$3(x-1)+3(y-3)+3(z-0)=0 \Rightarrow 3x+3y+3z-12=0$$

$$x+y+z-4=0$$



Note: You may check that if you choose a different point or a different normal vector (but which has to be parallel to $\vec{n}$), you will get the same equation.