

Math 603. Final Exam

(due by 5pm on Wednesday, May 5, 2010)

1. Let G be a finite group.
 - (a) Show that the set of isomorphism classes of one dimensional representations of G over a field K forms a group with a group law given by the tensor product of representations. This group is called the *Picard group* of G and is denoted by $\text{Pic}_K(G)$.
 - (b) Show that $\text{Pic}_{\mathbb{C}}(G)$ is isomorphic to the abelianization of G .
2. Find the Galois group of the polynomial $(x^2 - 2)(x^2 - 3)(x^2 - 5)$ over \mathbb{Q} .
3. Let $f : M \rightarrow N$ be a map of complexes of abelian groups.
 - (a) Show that the cone complex $\text{cone}(f)$ fits in a short exact sequence of complexes

$$0 \rightarrow N \rightarrow \text{cone}(f) \rightarrow M[1] \rightarrow 0.$$

- (b) Let $N \hookrightarrow C$ be an embedding of complexes and suppose that for all n , the map of abelian groups $N^n \hookrightarrow C^n$ is split, i.e. for each n we can choose an isomorphism

$$\varphi^n : C^n \xrightarrow{\cong} N^n \oplus (C/N)^n$$

compatible with the embedding of $N^n \subset C^n$ (but not necessarily compatible with the differentials). Set $M := (C/N)[-1]$ and use the splittings φ^n to write the differential d_C is a block form

$$d_C = \begin{pmatrix} d_N & f \\ 0 & d_{M[1]} \end{pmatrix}.$$

Show that $f : M \rightarrow N$ is a morphism of complexes and that the splitting isomorphisms $\varphi = (\varphi^\bullet)$ identify C with $\text{cone}(f)$.

4. Let X be a paracompact topological space (this means that X is Hausdorff and every open covering of X has a locally finite subcovering). A sheaf of abelian groups H on X is called *flabby* if for every open set $U \subset X$ it follows that $r_{XU} : H(X) \rightarrow H(U)$ is surjective. Now suppose F and G are sheaves of abelian groups on X , and G is an injective object in the abelian category $\mathbf{ABSh}(X)$ of abelian sheaves on X . Show that the sheaf H of all abelian sheaf homomorphisms from F to G is flabby.