Math 603. Homework 2
(due Wednesday, February 18, 2009)

1. Let S be an affine space over a field k, modelled on the vector space V. Let $f : S \to S$ be an affine map. Let $Df : V \to V$ be the unique linear map for which $f(s+v) = f(s) + Df(v)$ for all $v \in V$ and $s \in S$. The map Df is called the *differential* of f. Show that if 1 is not an eigenvalue of Df, then f has a fixed point.

2. Let S be an affine space over $\mathbb{R}$, and let $M \subset S$ be a convex subset. Show that if $s \notin M$, then the convex hull C of the subset $M \cup \{s\} \subset S$ is given as $C = \cup_{x \in M} \overline{sx}$, where $\overline{sx}$ denotes the convex hull of the points s and x.

3. Let S be a finite dimensional affine space over a field k. Show that the group $\mathrm{Aff}_k(S)$ of affine linear transformations of S admits a faithful representation in $GL_N(k)$ for some appropriately chosen $N > 0$.

4. Let S be a finite dimensional affine space over $\mathbb{R}$ and let M be a convex subset of S. Show that the closure $\overline{M}$ of M is also convex and that every interior point of $\overline{M}$ belongs to M.