Math 603. Homework 3

(due Wednesday, March 27, 2019)

1. **(a)** Let K be a field of characteristic p and let $a \in F$. Show that the polynomial $x^p - a \in F[x]$ is either irreducible or is the p-th power of a linear polinomial.

(b) Let P be a field of characteristic $p > 0$ and let $F = P(t)$ be the field of rational functions with coefficients in P. Show that the polynomial $x^p - t \in F[x]$ is irreducible.

2. Let $\Phi_n(x)$ be the n-th cyclotomic polynomial. Show that for every $d < n$ with $d|n$ the product $(x^d - 1)\Phi_n(x)$ divides $x^n - 1$ in the polynomial ring $\mathbb{Z}[x]$.

3. Consider the tower of finite fields

$$\mathbb{F}_p \subset \mathbb{F}_{p^{2!}} \subset \mathbb{F}_{p^{3!}} \subset \mathbb{F}_{p^{4!}} \subset \cdot.$$

Let $\mathbb{F}_{p^{\infty!}} = \cup_{n=1}^{\infty} \mathbb{F}_{p^{n!}}$ with the natural addition and multiplication. Show that $\mathbb{F}_{p^{\infty!}}$ is algebraically closed.