

Math 603. Homework 4

(due Wednesday, March 18, 2009)

- (a) Let K be a field of characteristic p and let $a \in F$. Show that the polynomial $x^p - a \in F[x]$ is either irreducible or is the p -th power of a linear polynomial.

(b) Let P be a field of characteristic $p > 0$ and let $F = P(t)$ be the field of rational functions with coefficients in P . Show that the polynomial $x^p - t \in F[x]$ is irreducible.
- Let $\Phi_n(x)$ be the n -th cyclotomic polynomial. Show that for every $d < n$ with $d|n$ the product $(x^d - 1)\Phi_n(x)$ divides $x^n - 1$ in the polynomial ring $\mathbb{Z}[x]$.
- Consider the tower of finite fields

$$\mathbb{F}_p \subset \mathbb{F}_{p^{2!}} \subset \mathbb{F}_{p^{3!}} \subset \mathbb{F}_{p^{4!}} \subset \dots$$

Let $\mathbb{F}_{p^\infty} = \bigcup_{n=1}^{\infty} \mathbb{F}_{p^{n!}}$ with the natural addition and multiplication. Show that \mathbb{F}_{p^∞} is algebraically closed.