Exercise 1. Given a triangle $\Delta ABC$, and a point $P$ inside the triangle, we denote by $d_a, d_b, d_c$ the distances from $P$ to the lines which contain the sides of the triangle. Find the point $P$ for which the product $d_a \cdot d_b \cdot d_c$ is maximized.

Exercise 2. Suppose that $x, y, z > 0$. Show that:

$$\frac{x^3}{x^2 + xy + y^2} + \frac{y^3}{y^2 + yz + z^2} + \frac{z^3}{z^2 + zx + x^2} \geq \frac{x + y + z}{3}$$

Furthermore, show that equality holds if and only if $x = y = z$.

Exercise 3. Let $I_n := 1 + \frac{1}{2} + \cdots + \frac{1}{n}$. Without using the logarithmic asymptotics for $I_n$, show that:

$$I_n \geq n(\sqrt[n]{n} + 1 - 1).$$