NAME:

114 Spring 2011: Calculus

MIDTERM I : PRACTICE!

This practice exam is *not* multiple choice. Just work out these problems. Several of these problems come from old 114 exams. The length of this exam will be a full 80 minutes: be prepared to do lots of problems. Most will be on "the basic" but a few will dig deeper. Note that you will likely see problems on the real midterm which look nothing like what you see here!

PROBLEM 1: Write the equation of the plane that contains both the lines

$$L_1 = \begin{pmatrix} 3-t \\ -4+t \\ 4+2t \end{pmatrix} \quad ; \quad L_2 = \begin{pmatrix} 3+s \\ -4+s \\ 4-s \end{pmatrix}$$

PROBLEM 2: What is the radius of the *smallest* osculating circle tangent to the parabola $y = x^2$? Can you write out its equation?

PROBLEM 3: Compute the arclength of a helix $\vec{r}(t) = \sin t\hat{\imath} - 2t\hat{\jmath} + \cos t\hat{k}$ for $\pi \leq t \leq 2\pi$.

PROBLEM 4: For which values of k are the following vectors in 4-dimensional space orthogonal?

$$\vec{u} = \begin{pmatrix} 1 \\ -2 \\ k \\ 0 \end{pmatrix} \quad ; \quad \vec{v} = \begin{pmatrix} k^2 \\ 3 \\ -1 \\ 2 \end{pmatrix}$$

PROBLEM 5: Compute the derivative of the function F that returns the volume V and surface area A of a solid cylinder of as a function of its radius r and length ℓ. Your answer should be in the form of a matrix.

PROBLEM 6: What is the area of the parallelogram spanned by the vectors

$$\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \quad ; \quad \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$$

PROBLEM 7: A point moves on the hyperbola $x^2 - y^2 = 1$ with position vector

$$\vec{r}(t) = \cosh(\omega t)\hat{\imath} + \sinh(\omega t)\hat{\jmath}$$

where ω is a constant. Show that the acceleration vector $\vec{a}$ is a scalar multiple of $\vec{r}$. What is that scalar factor?

PROBLEM 8: Show that the points $(0,0,0)$, $(1,1,0)$, $(1,0,1)$, and $(0,1,1)$ are the corners of a regular tetrahedron, by showing that all vectors between them have the same length. What is the angle between any two intersecting edges of a regular tetrahedron? [This is on some importance in chemistry — it tells you about molecule geometry.]

PROBLEM 9: A rectangular box with dimensions height h, width w, and depth d, costs 5 dollars per square inch for the top, 8 dollars per square inch on the bottom, and 3 dollars per square inch for all sides. When $h = w = d = 4$, what is the derivative of the function recording the (1) volume, (2) area, and (3) cost of the box? If you want to increase the volume and area with the smallest possible increase in cost, which dimension(s) would you increase? If h increases at rate 2, w decreases at rate 3, and h remains constant, at what rate do these three outputs change?

PROBLEM 10: Two particles move through space along the curves

$$\vec{r} = \begin{pmatrix} 2+t \\ -2+2t \\ 3-t \end{pmatrix} \quad ; \quad \vec{s} = \begin{pmatrix} 2-4t+t^2 \\ -2+7t-t^2 \\ 3-6t+t^2 \end{pmatrix}$$

Compute the time $t > 0$ at which these particles intersect, and compute their unit tangent and unit normal vectors and curvatures at the intersection point. (Note: this will be an ugly answer...)

PROBLEM 11: Compute the derivative of the *spherical coordinates* map:

$$S\begin{pmatrix} \rho \\ \theta \\ \phi \end{pmatrix} = \begin{pmatrix} \rho\cos\theta\sin\phi \\ \rho\sin\theta\sin\phi \\ \rho\cos\phi \end{pmatrix}$$

What is the determinant of this matrix?

PROBLEM 12: The ideal gas law says $PV = nRT$ (as you know). Show explicitly that

$$\frac{\partial P}{\partial V}\frac{\partial V}{\partial T}\frac{\partial T}{\partial P} = -1.$$